Name: Student ID:

Quiz #1 5%

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

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This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (1.5%) Answer the following questions:
   a) In how many ways can the letters in DATAGRAM be arranged?
   b) For the arrangement of part (a), how many have all three A's together?
   c) In how many ways can the letters in SOCIOLOGICAL be arranged with all the vowels adjacent?

   Answer:

   a) *Since there are three A's, there are* $\frac{8!}{3!}$ *arrangements*
   b) *Consider three A's together as a symbol. Hence, we have six symbols D,T,G,R,M,(AAA). There are* $6!$ *arrangements*
   c) *Consider an example where all the vowels are adjacent: S,C,L,G,C,L,(OOOIIA). Seven symbols here can be arranged in* $\frac{7!}{2!\,2!}$ *ways. (OOOIIA) can be arranged in* $\frac{6!}{3!\,2!}$ *ways. Hence, there are* $\frac{7!}{2!\,2!} \times \frac{6!}{3!\,2!}$*arrangements where all the vowels are adjacent*

2) *(1.5%) Determine the coefficient of* $w^2x^2y^2z^2$ *in the expansion of*
   a) $(w + x + y + z + 1)^2$
   b) $(2w - x + 3y + z - 2)^12$
   c) $(v + w - 2x + y + 5z + 3)^12$

   Answer:

   a) 0
   b) $\binom{12}{2,2,2,2,4}(2)^2(-1)^2(3)^2(1)^2(-2)^4 = \frac{12!}{2!^4\,4!} \times (2)^2(3)^2(2)^4$
   c) $\binom{12}{0,2,2,2,2,4}(1)^2(-2)^2(1)^2(5)^2(3)^4 = \frac{12!}{2!^4\,4!} \times (2)^2(5)^2(3)^4$

3) (2%) Consider the strings made up of n bits – that is, a total of n 0's and 1's. In particular consider those strings with exactly four occurrences of 01. For $n \geq 8$, How many such

strings are there?

Answer:

For $n \geq 8$*, a string with this structure has* x_1 *1's followed by* x_2 *0's followed by* x_3 *1's ... followed by* x_{10} *0's, where* $x_1 + x_2 + ... + x_{10} = n$*,* $x_1, x_{10} \geq 0$*,* $x_2, ..., x_9 > 0$*.*

The number of solutions to this equation equals to the number of solutions to $y_1 + y_2 + ... + y_{10} = n - 8$*. The number of this equation is* $\binom{10+(n-8)-1}{n-8} = \binom{n+1}{9}$