Name: Student ID:

Quiz #2 5%

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This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (2%) For primitive statements p, q, r, and s
   a) Simplify the compound statement $[[[(p \wedge q) \wedge r] \vee [(p \wedge q) \wedge \neg r]] \vee \neg q] \rightarrow s$ to a statement without $\neg$, $\vee$, and $\wedge$.
   b) Verify that $[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)] \Leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)]$

Answer:

a) *(1)*$[(p \wedge q) \wedge r] \vee [(p \wedge q) \wedge \neg r] \Leftrightarrow (p \wedge q) \wedge (r \vee \neg r) \Leftrightarrow (p \wedge q) \wedge T_0 \Leftrightarrow p \wedge q$, *where* T_0 *represents tautology in our expression.*

(2) $(p \wedge q) \vee \neg q \Leftrightarrow (p \vee \neg q) \wedge (q \vee \neg q) \Leftrightarrow (p \vee \neg q) \wedge T_0 \Leftrightarrow (p \vee \neg q)$

(3) $(p \vee \neg q) \rightarrow s \Leftrightarrow (q \rightarrow p) \rightarrow s$

Therefore, the given statement can be simplified to $(q \rightarrow p) \rightarrow s$.

b)

p	q	r	$(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)$	$(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)$
0	0	0	1	1
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

2) *(1%) Proof the following argument is valid.*

p

$(p \rightarrow q)$

$r \rightarrow \neg q$

$s \vee r$

Therefore, $s \vee t$

Answer:

Steps

a) p //Premise
b) $(p \rightarrow q)$ //Premise
c) q // Steps (a), (b), and the Rule of Detachment
d) $r \rightarrow \neg q$ //Premise
e) $q \rightarrow \neg r$ // Step (d) and $(r \rightarrow \neg q) \Leftrightarrow (\neg\neg q \rightarrow \neg r) \Leftrightarrow (q \rightarrow \neg r)$
f) $\neg r$ // Steps (c), (e), and the Rule of Detachment
g) $s \vee r$ // Premise
h) s // Steps (f), (g), and the Rule of Disjunctive Syllogism
i) Therefore, $s \vee t$ //Step (h) and the Rule of Disjunctive Amplification

3) (1%) Let $p(x), q(x)$ denote the following open statements.

   $p(x) : x \leq 3, q(x) : x + 1$ is odd

   If the universe consists of all integers, what are the truth values?

   a) $q(1)$
   b) $\neg p(3)$
   c) $p(7) \vee q(7)$
   d) $p(3) \wedge q(4)$
   e) $\neg(p(-4) \vee q(-3))$
   f) $\neg p(-4) \wedge \neg q(-3)$

   Answer:

   a) *False*
   b) *False*
   c) *False*
   d) *True*
   e) *False*
   f) *False*

4) *(1%) Let n be an integer. Prove that n is odd if and only if $7n + 8$ is odd.*

   Answer:

   If n is odd, then $n = 2k+1$ for some (particular) integer k. Then $7n+8 = 7(2k+1)+8 = 14k + 15 = 14k + 14 + 1 = 2(7k + 7) + 1$, so it follows from Definition 2.8 that $7n + 8$ is odd. Conversely, suppose that n is not odd. Then n is even, so $n = 2t$ for some (particular) integer t. But then $7n + 8 = 7(2t) + 8 = 14t + 8 = 2(7t + 4)$, so it follows from Definition 2.8 that $7n + 8$ is even – that is, $7n + 8$ is not odd. Consequently, the converse follows by contraposition.

   Definition 2.8 : Let n be an integer. We call n even if n is divisible by 2 – that is if there exists an integer r so that $n = 2r$. If n is not even, then we call n odd and find for this case that there exists an integer s where $n = 2s + 1$