Name: Student ID:

Quiz #3 5%

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

Department of Computing Science, National Tsing Hua University, Taiwan

This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (1.5%) For $A = \{1, 2, 3, 4, 5, 6, 7\}$, determine the number of
   a) subsets of A
   b) nonempty proper subsets of A
   c) subsets of A containing three elements
   d) subsets of A containing five elements, including 1 and 2
   e) subsets of A containing with an odd number of elements

   Answer:
   a) 2^7
   b) 126
   c) $\binom{7}{3}$
   d) $\binom{5}{3}$
   e) $\binom{7}{1} + \binom{7}{3} + \binom{7}{5} + \binom{7}{7} = 64$

2) *(1%) For a given universe $\mathscr{U}$, let $A \subseteq \mathscr{U}$ where A is finite with $|\mathscr{P}(A)| = n$. If $B \subseteq \mathscr{U}$, how many subsets does B have, if*
   a) $B = A \cup \{x\}$, *where* $x \in \mathscr{U} - A$ *?*
   b) $B = A \cup \{x, y\}$, *where* $x, y \in \mathscr{U} - A$ *?*
   c) $B = A \cup \{x_1, x_2, x_3, \ldots, x_k\}$, *where* $x_1, x_2, x_3, \ldots, x_k \in \mathscr{U} - A$ *?*

   Answer:
   a) $2n$
   b) $2^2 n = 4n$
   c) $2^k n$

3) (1%) If $A = \{a, b, d\}$, $B = \{d, x, y\}$, and $C = \{x, z\}$, how many proper subsets are there for the set

   a) $(A \cap B) \cup C$?

   b) $A \cap (B \cup C)$?

   Answer:

   a) $(A \cap B) \cup C = \{d, x, z\}$ *which has* $2^3 - 1 = 7$ *proper sets.*

   b) $A \cap (B \cup C) = \{d\}$ *which has 1 proper set.*

4) *(2%) How many permutations of the 26 different letters of the alphabet contain*

   a) *either the pattern "OUT" or the pattern "DIG" ?*

   b) *neither the pattern "MAN" nor the pattern "NAT" ?*

   Answer:

   a) There are $24!$ permutations containing each of thje patterns "OUT" and "DIG". There are $22!$ permutations containing both patterns. Hence, there are $2 \times 24! - 22!$ permutations containing either the pattern "OUT" or the pattern "DIG".

   b) There are $26!$ permutations in total. There are $24!$ permutations that contain each of the pattern "MAN" and "NAT", and $23!$ that contain both (i.e. "MANT"). Hence, there are $2 \times 24! - 23!$ permutations that contain either "MAN" or "NAT", and $26! - (2 \times 24! - 23!)$ permutations that contain neither pattern.