Name: Student ID:

Quiz #6 5+2%

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This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (2%) For $\Sigma = \{0, 1\}$ describe the strings in A^* for each of the following language $A \subseteq \Sigma^*$
   a) $\{01\}$
   b) $\{000\}$
   c) $\{0, 010\}$
   d) $\{1, 10\}$

   Solution:
   a) Here A^* consist of all strings x of even length where if $x \neq \lambda$ then x starts with 0 and ends with 1. The symbols $(0$ and $1)$ can be alternative.
   b) In this case A^* contains strings made up of $3n$ 0's for $n \in N$
   c) Here a string $x \in A^*$ if and only if

      1) x is a string of n 0's for $n \in N$

      2) x is a string that starts and ends with 0, and it has at least one 1 without consecutive 1's
   d) For this case A^* consist of the following:

      1) Any string of n 1's for $n \in N$

      2) Any string that starts with 1 and contain at least one 0 without consecutive 0's

2) (1%) Show that it is not possible to construct a finite state machine which recognizes precisely those sequences in the language $A = \{0^i 1^j | i,\ j \in Z^+, i > j\}$. Here the alphabet for A is $\Sigma = \{0, 1\}$

   Solution:

   Suppose the contrary and let the machine have n states, for some $n \in Z^+$. Consider the input string $0^{n+1}1^n$. We expect the output here to be $0^{n+1}1^n$. As the 0's in this input string are processed we obtain $n+1$ states $s_1, s_2, s_3, \cdots s_n s_{n+1}$ from the function ν. By the Pigeonhole Principle, there are two states s_i, s_j where $i < j$ but $s_i = s_j$. Hence, if the states s_m where $i+1 \leq m \leq j$ are removed and it's along with their inputs of 0, then this machine will recognize the sequence $0^{n+1-(j-i)}1^n$ where $n+1-(j-i) \leq n$. However, this string $0^{n+1-(j-i)}1^n \notin A$

3) (2%) Let $M = \{S, \mathscr{I}, \mathscr{O}, \nu, \omega\}$ be a finite state machine with $\mathscr{I} = \mathscr{O} = \{0, 1\}$. S, ν, and ω are determined by the state diagram shown in Fig. 1

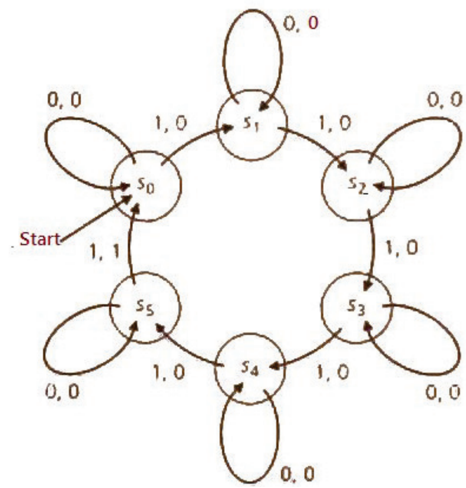

Fig. 1. State Diagram

a) Find the output for the input string x=0110111011
b) Give the transition table for this finite state machine
c) Starting in state s_0, if the output for an input string x is 0000001. Please show all possibilities for x
d) Describe in words what this finite machine does

Solution:

a) 0000000010

b)

	ν	ν	ω	ω
	0	1	0	0
s_0	s_0	s_1	0	0
s_1	s_1	s_2	0	0
s_2	s_2	s_3	0	0
s_3	s_3	s_4	0	0
s_4	s_4	s_5	0	0
s_5	s_5	s_0	0	1

c) $\omega(x, s_0) = 0000001$ for $x =$

i) 1111101

ii) 1111011

iii) 1110111

iv) 1101111

v) 1011111

vi) 0111111

vii) 0111111

d) The machine recognizes the occurrence of a sixth 1, a 12^{th} 1 in an input x