Name: Student ID:

Quiz #8 5%

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

Department of Computing Science, National Tsing Hua University, Taiwan

This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (1%) There are 3 red marbles, 3 blue marbles, 3 white marbles, and 3 green marbles. In how may ways can John select nine marbles from a bag of twelve (identical except for color) ?

   Solution: The answer is the number of integer solutions for $x_1 + x_2 + x_3 + x_4 = 9$, $0 \leq x_i \leq 3$, $1 \leq i \leq 4$. Let c_i denote a solution that $x_i \geq 4$ for $1 \leq i \leq 4$. Then, $N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4}) = \binom{12}{9} - \binom{4}{1}\binom{8}{5} + \binom{4}{2}\binom{4}{1}$

2) (1%) Let $A = \{1, 2, 3, \cdots, 10\}$, $B = \{1, 2, 3, \cdots, 7\}$, How many functions $f : A \to B$ satisfy $|f(A)| = 4$?

   Solution: For $1 \leq i \leq 7$, let c_i denote the condition that i is not in the range of f. Then the number of functions $f : A \to B$ where $|f(A)| = 4$ is $E_3 = S_3 - \binom{4}{1}S_4 + \binom{5}{2}S_5 - \binom{6}{3}S_6 + \binom{7}{4}S_7 = \binom{7}{3}4^{10} + \binom{4}{1}\binom{7}{4}3^{10} + \binom{5}{2}\binom{7}{5}2^{10} - \binom{6}{3}\binom{7}{6}1^{10} + \binom{7}{4}\binom{7}{7}0^{10}$

3) (1%) For the positive integers $1, 2, 3 \cdots, n-1, n$, there are 11660 dearrangements where $1, 2, 3, 4,$ and 5 appear in the first five positions. What's the value of n ?

   Solution: Let $n = m + 5$. Then $11660 = d_5 \cdot d_m = 44d_m$. n equals to 11 since $d_m = 265 = d_6$.

4) (2%) Find the rook polynomials for the shaded chessboards in Fig. 1.

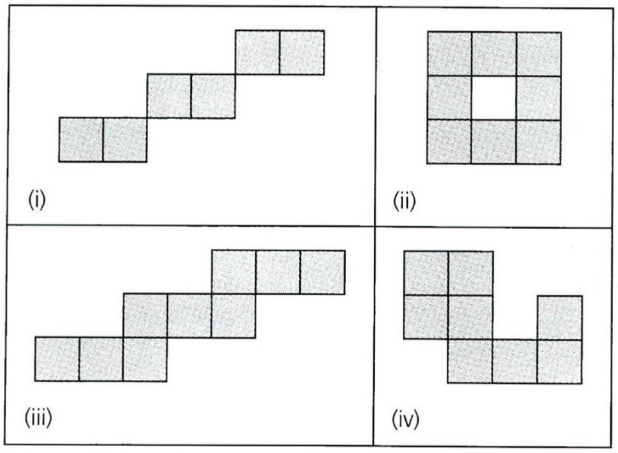

Fig. 1. The chessboards.

Solution:

i) $1 + 6x + 12x^2 + 8x^3$

ii) $1 + 8x + 14x^2 + 4x^3$

iii) $1 + 9x + 25x^2 + 21x^3$

iv) $1 + 8x + 16x^2 + 7x^3$