Chapter5 Solutions

Ex 5.1: 1, 3, 6, 8, 12

Ex 5.2: 4, 8, 15, 20, 27

Ex 5.3: 1, 4, 8, 12, 16

Ex 5.4: 1, 2, 5, 8, 12

Ex 5.5: 2, 6, 13, 14, 20

Ex 5.6: 7, 10, 16, 17, 22

Ex 5.1: (1)

1. If $A = \{1, 2, 3, 4\}$, $B = \{2, 5\}$, and $C = \{3, 4, 7\}$, determine $A \times B$; $B \times A$; $A \cup (B \times C)$; $(A \cup B) \times C$; $(A \times C) \cup (B \times C)$.

- $A \times B =$
  $\{(1,2), (2,2), (3,2), (4,2), (1,5), (2,5), (3,5), (4.5)\}$
- $B \times A =$
  $\{(2.1), (2.2), (2.3), (2,4), (5,1), (5,2), (5,3), (5,4)\}$
- $A \cup (B \times C) =$
  $\{1,2,3,4, (2,3), (2,4), (2,7), (5,3), (5,4), (5,7)\}$
- $(A \cup B) \times C = (A \times C) \cup (B \times C) =$
  $\{(1,3), (2,3), (3,3), (4,3), (5,3),$
  $(1,4), (2,4), (3,4), (4,4), (5,4),$
  $(1,7), (2,7), (3,7), (4,7), (5,7)\}$

Ex 5.1: (3)

3. For A, B as in Exercise 2, determine the following: (a) $|A \times B|$; (b) the number of relations from A to B; (c) the number of relations on A; (d) the number of relations from A to B that contain (1, 2) and (1, 5); (e) the number of relations from A to B that contain exactly five ordered pairs; and (f) the number of relations on A that contain at least seven elements.

a) $|A \times B| = |A||B| = 9$

b) 2^9

c) 2^9

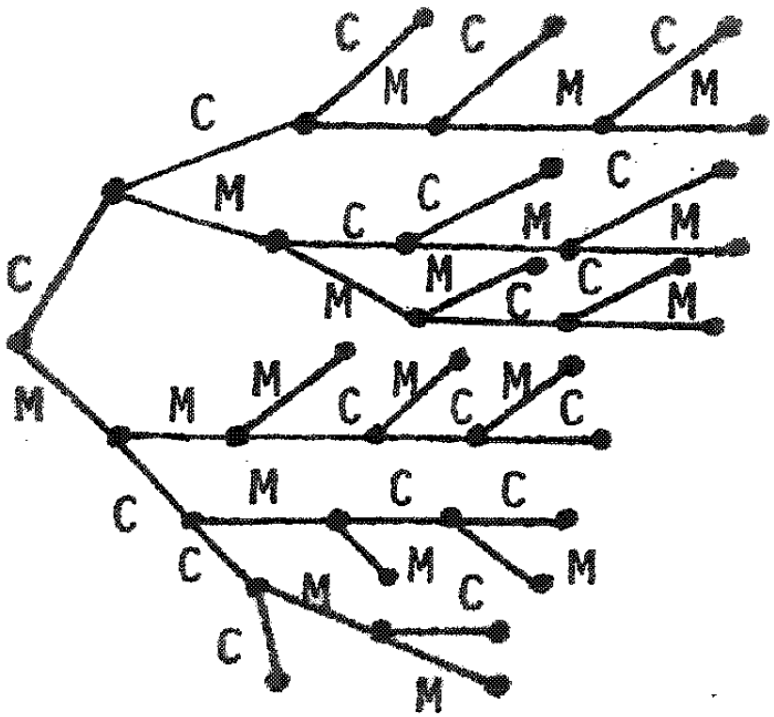
d) 2^7

e) $\binom{9}{5}$

f) $\binom{9}{7} + \binom{9}{8} + \binom{9}{9}$

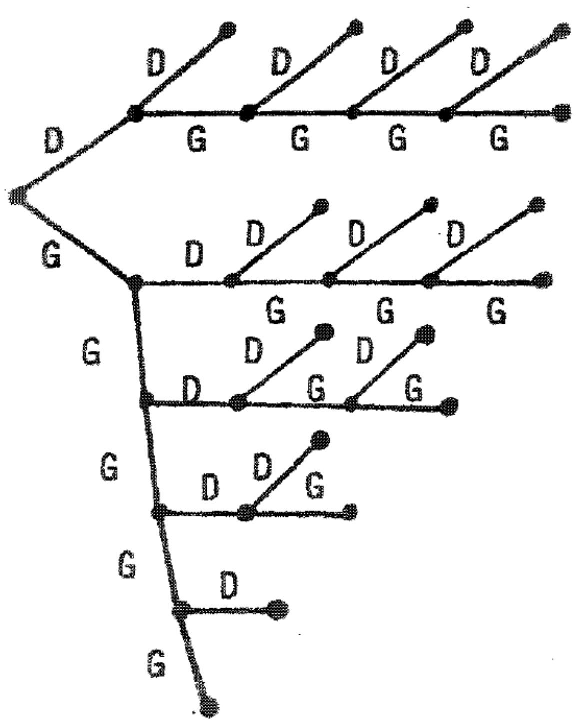
Ex 5.1: (6)

6. The men's final at Wimbledon is won by the first player to win three sets of the five-set match. Let C and M denote the players. Draw a tree diagram to show all the ways in which the match can be decided.



Ex 5.1: (8)

8. Logic chips are taken from a container, tested individually, and labeled defective or good. The testing process is continued until either two defective chips are found or five chips are tested in total. Using a tree diagram, exhibit a sample space for this process.



Ex 5.1: (12)

12. Let A, B be sets with $|B| = 3$. If there are 4096 relations from A to B, what is $|A|$?

- $2^{3|B|} = 4096 \Longrightarrow 3|B| = 12 \Longrightarrow |B| = 4.$

Ex 5.2: (4)

4. If there are 2187 functions $f\colon A \to B$ and $|B| = 3$, what is $|A|$?

- $3^{|A|} = 2187 \Longrightarrow |A| = 7.$

Ex 5.2: (8)

8. Determine whether each of the following statements is true or false. If the statement is false, provide a counterexample.

a) $\lfloor a \rfloor = \lceil a \rceil$ for all $a \in \mathbf{Z}$.

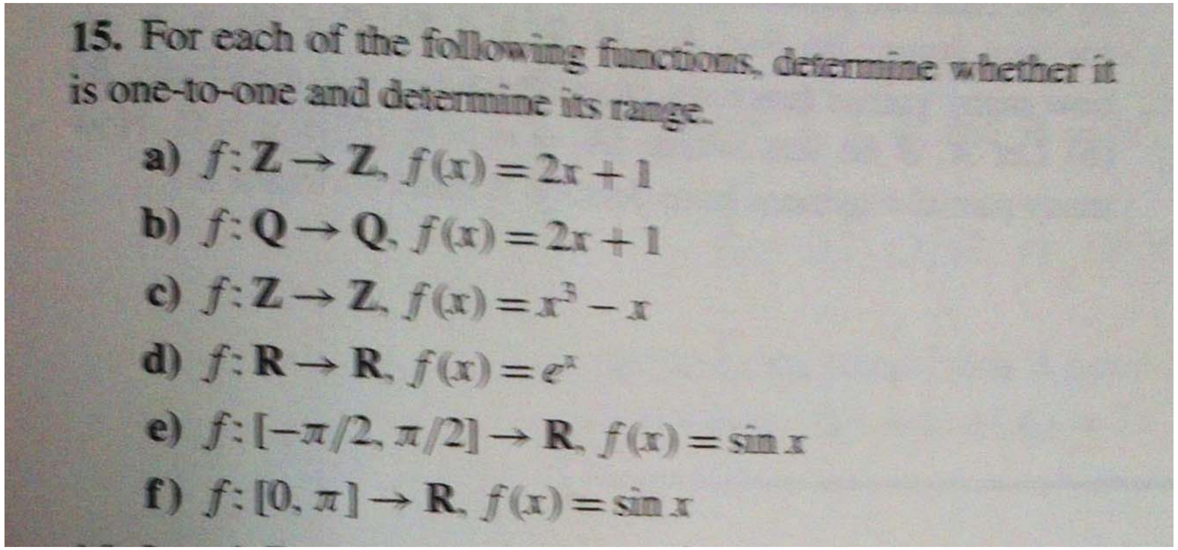
b) $\lfloor a \rfloor = \lceil a \rceil$ for all $a \in \mathbf{R}$.

c) $\lfloor a \rfloor = \lceil a \rceil - 1$ for all $a \in \mathbf{R} - \mathbf{Z}$.

d) $-\lceil a \rceil = \lceil -a \rceil$ for all $a \in \mathbf{R}$.

a) True
b) False: Let $a = 1.5$. Then $\lfloor 1.5 \rfloor = 1 \neq 2 = \lceil 1.5 \rceil$
c) True
d) False: Let $a = 1.5$. Then $-\lceil a \rceil = -2 \neq -1 = \lceil -a \rceil$

Ex 5.2: (15)



15. For each of the following functions, determine whether it is one-to-one and determine its range.

a) $f: \mathbf{Z} \to \mathbf{Z}$, $f(x) = 2x + 1$

b) $f: \mathbf{Q} \to \mathbf{Q}$, $f(x) = 2x + 1$

c) $f: \mathbf{Z} \to \mathbf{Z}$, $f(x) = x^3 - x$

d) $f: \mathbf{R} \to \mathbf{R}$, $f(x) = e^x$

e) $f: [-\pi/2, \pi/2] \to \mathbf{R}$, $f(x) = \sin x$

f) $f: [0, \pi] \to \mathbf{R}$, $f(x) = \sin x$

a) One-to-one. The range is the set of all odd integers.

b) One-to-one. Range = Q.

c) Since $f(1) = f(0)$, f is not one-to-one. The range of $f = \{0, \pm 6, \pm 24, \pm 60, \ldots\} = \{n^3 - n | n \in \mathbb{Z}\}$.

d) One-to-one. Range = $(0, +\infty) = \mathbb{R}^+$.

e) One-to-one. Range = $[-1,1]$.

f) Since $f(\frac{\pi}{4}) = f(\frac{3\pi}{4})$, f is not one-to-one. The range of $f = [0,1]$.

Ex 5.2: (20)

20. If $A = \{1, 2, 3, 4, 5\}$ and there are 6720 injective functions $f: A \to B$, what is $|B|$?

- The number of injective (or, one-to-one) functions from A to B is $(|B|!)/(|B| - 5)! = 6720$, and $|B| = 8$.

Ex 5.2: (27)

27. One version of *Ackermann's function A(m,n)* is defined recursively for $m, n \in \mathbf{N}$ by

$$A(0, n) = n + 1, n \geq 0;$$
$$A(m, 0) = A(m - 1, 1), m > 0; \text{ and}$$
$$A(m, n) = A(m - 1, A(m, n - 1)), m, n > 0.$$

[Such functions were defined in the 1920s by the German mathematician and logician Wilhelm Ackermann (1896–1962), who was a student of David Hilbert (1862–1943). These functions play an important role in computer science — in the theory of recursive functions and in the analysis of algorithms that involve the union of sets.]

a) Calculate $A(1, 3)$ and $A(2, 3)$.

b) Prove that $A(1, n) = n + 2$ for all $n \in \mathbf{N}$.

c) For all $n \in \mathbf{N}$ show that $A(2, n) = 3 + 2n$.

d) Verify that $A(3, n) = 2^{n+3} - 3$ for all $n \in \mathbf{N}$.

Ex 5.2: (27.a)

$$A(1,3) = A(0,A(1,2)) = A(1,2) + 1 = A(0,A(1,1)) + 1$$
$$= [A(1,1) + 1] + 1 = A(1,1) + 2 = A(0,A(1,0)) + 2$$
$$= [A(1,0) + 1] + 2 = A(1,0) + 3 = A(0,1) + 3 = (1 + 1) + 3$$
$$= 5$$
$$A(2,3) = A(1,A(2,2))$$
$$A(2,2) = A(1,A(2,1))$$
$$A(2,1) = A(1,A(2,0)) = A(1,A(1,1))$$
$$A(1,1) = A(0,A(1,0)) = A(1,0) + 1 = A(0,1) + 1 = (1 + 1) + 1 = 3$$
$$A(2,1) = A(1,3) = A(0,A(1,2)) = A(1,2) + 1 = A(0,A(1,1))$$
$$= [A(1,1) + 1] + 1 = 5$$
$$A(2,2) = A(1,5) = A(0,A(1,4)) = A(1,4) + 1 = A(0,A(1,3)) + 1$$
$$= A(1,3) + 2 = A(0,A(1,2)) + 2 = A(1,2) + 3$$
$$= A(0,A(1,1)) + 3 = A(1,1) + 4 = 7$$
$$A(2,3) = A(1,7) = A(0,A(1,6)) = A(1,6) + 1 = A(0,A(1,5)) + 1$$
$$= A(0,7) + 1 = (7 + 1) + 1 = 9$$

Ex 5.2: (27.b)

Since $A(1,0) = A(0,1) = 2 = 0 + 2$, the result holds for the case where $n = 0$. Assuming the truth of the (open) statement for some k (≥ 0) we have $A(1, k) = k + 2$. Then we find that $A(1, k + 1) = A(0, A(1, k)) = A(1, k) + 1 = (k + 2) + 1 = (k + 1) + 2$, so the truth at $n = k$ implies the truth at $n = k + 1$. Consequentiy, $A(1, n) = n + 2$ for all $n \in \mathrm{N}$ by the Principle of Mathematical Induction.

Ex 5.2: (27.c)

Here we find that $A(2,0) = A(1,1) = 1 + 2 = 3$ (by the result in part(b)). So $A(2,0) = 3 + 2 \cdot 0$ and the given (open) statement is true in this first case. Next we assume the result true for some $k(\geq 0)$ - that is, we assume that $A(2,k) = 3 + 2k$. For $k + 1$ we then find that $A(2,k+1) = A(1,A(2,k)) = A(2,k) + 2$ (by part (b))= $(3 + 2k) + 2$ (by the induction hypothesis)= $3 + 2(k + 1)$. Consequently, for all $n \in \mathrm{N}, A(2,n) = 3 + 2n$ - by the Principle of Mathematical Induction.

Ex 5.2: (27.d)

Once again we consider what happens for n=0. Since $A(3,0) = A(2,1) = 3 + 2(1)$ (by part (c)) $= 5 = 2^{0+3} - 3$, the result holds in this first case. So now we assume the given (open) statement is true for some k (≥ 0) and this gives us the induction hypothesis: $A(3,k) = 2^{k+3} - 3$. For $n = k + 1$ it then follows that $A(3,k+1) = A(2,A(3,k)) = 3 + 2A(3,k)$ (by part (c))$= 3 + 2(2^{k+3} - 3)$ (by the induction hypothesis)$= 2^{(k+1)+3} - 3$, so the result holds for $n = k + 1$ whenever it does for $n = k$. Therefore, $A(3,n) = 2\text{^}(n+3) - 3$, for all $n \in$ N - by the Principle of Mathematical Induction.

Ex 5.3: (1)

1. Give an example of finite sets A and B with $|A|, |B| \geq 4$ and a function $f: A \rightarrow B$ such that (a) f is neither one-to-one nor onto; (b) f is one-to-one but not onto; (c) f is onto but not one-to-one; (d) f is onto and one-to-one.

- Let $A = \{1,2,3,4\}, B = \{v, w, x, y, z\}$:

a) $f = \{(1, v), (2, v), (3, w), (4, x)\}$

b) $f = \{(1, v), (2, x), (3, y), (4, z)\}$

c) Let $A = \{1,2,3,4,5\}, B = \{w, x, y, z\}$,
$f = \{(1, w), (2, w), (3, x), (4, y), (5, z)\}$.

d) Let $A = \{1,2,3,4\}, B = \{w, x, y, z\}$,
$f = \{(1, w), (2, x), (3, y), (4, z)\}$

Ex 5.3: (4)

4. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$. (a) How many functions are there from A to B? How many of these are one-to-one? How many are onto? (b) How many functions are there from B to A? How many of these are onto? How many are one-to-one?

a) $6^4; \ \frac{6!}{2!}; \ 0$

b) $4^6; (4!)S(6,4); 0$

Ex 5.3: (8)

8. A chemist who has five assistants is engaged in a research project that calls for nine compounds that must be synthesized. In how many ways can the chemist assign these syntheses to the five assistants so that each is working on at least one synthesis?

- Let A be the set of compounds and B the set of assistants. Then the number of assignments with no idle assistants is the number of onto functions from set A to set B. There are $5!\,S(9,5)$ such functions.

Ex 5.3: (12)

12. a) In how many ways can 31,100,905 be factored into three factors, each greater than 1, if the order of the factors is not relevant?

b) Answer part (a), assuming the order of the three factors is relevant.

c) In how many ways can one factor 31,100,905 into two or more factors where each factor is greater than 1 and no regard is paid to the order of the factors?

d) Answer part (c), assuming the order of the factors is to be taken into consideration.

Ex 5.3: (12)

a) Since $31{,}100{,}905 = 5 \times 11 \times 17 \times 29 \times 31 \times 37$, we find that there are $S(6,3) = 90$ unordered factorizations of 31,100,905 into three factors - each greater than 1.

b) If the order of the factors in part (a) is considered relevant then there are $(3!)S(6,3) = 540$ such factorizations.

c) $\sum_{i=2}^{6} S(6,i) = S(6,2) + S(6,3) + S(6,4) + S(6,5) + S(6,6) = 202$

d) $\sum_{i=2}^{6} (i!)S(6,i) = (2!)S(6,2) + (3!)S(6,3) + (4!)S(6,4) + (5!)S(6,5) + (6!)S(6,6) = 4682$

Ex 5.3: (16)

16. At St. Xavier High School ten candidates $C_1, C_2, \ldots, C_{10}$, run for senior class president.

a) How many outcomes are possible where (i) there are no ties (that is, no two, or more, candidates receive the same number of votes? (ii) ties are permitted? [Here we may have an outcome such as $\{C_2, C_3, C_7\}, \{C_1, C_4, C_9, C_{10}\}, \{C_5\}, \{C_6, C_8\}$, where C_2, C_3, C_7 tie for first place, C_1, C_4, C_9, C_{10} tie for fourth place, C_5 is in eighth place, and C_6, C_8 are tied for ninth place.] (iii) three candidates tie for first place (and other ties are permitted)?

b) How many of the outcomes in section (iii) of part (a) have C_3 as one of the first-place candidates?

c) How many outcomes have C_3 in first place (alone, or tied with others)?

Ex 5.3: (16)

a) (i) 10!
   (ii) The given outcome - namely, $\{C_2, C_3, C_7\}, \{C_1, C_4, C_9, C_{10}\}, \{C_5\}, \{C_6, C_8\}$ - is an example of a distribution of ten distinct objects among four distinct containers, with no container left empty. [Or it is an example of an onto function $f: A \to B$ where $A = \{C_1, C_2, \ldots, C_{10}\}$ and $B = \{1,2,3,4\}$.] There are $4!\,S(10,4)$ such distributions [or functions].
   The answer to the question is $\sum_{i=1}^{10} i!\,S(10,i)$.
   (iii) $\binom{10}{3} \sum_{i=1}^{7} i!\,S(7,i)$.

b) $\binom{9}{2} \sum_{i=1}^{7} i!\,S(7,i)$.

c) For $0 \le k \le 9$, the number of outcomes where C_3 is tied for first place with k other candidates is $\binom{9}{k} \sum_{i=1}^{9-k} i!\,S(9-k,i)$. [Part (b) above is the special case where $k = 3 - 1 = 2$.] Summing over the possible values of k we have the answer $\sum_{k=0}^{9} \binom{9}{k} \sum_{i=1}^{9-k} i!\,S(9-k,i)$

Ex 5.4: (1)

1. For $A = \{a, b, c\}$, let $f: A \times A \to A$ be the closed binary operation given in Table 6. Give an example to show that f is *not* associative.

Table 6

f	a	b	c
a	b	a	c
b	a	c	b
c	c	b	a

- Here we find, for example, that $f(f(a,b),c) = f(a,c) = c$, while $f(a,f(b,c)) = f(a,b) = a$, so f is not associative.

Ex 5.4: (2)

2. Let $f: \mathbf{R} \times \mathbf{R} \to \mathbf{Z}$ be the closed binary operation defined by $f(a, b) = \lceil a + b \rceil$. (a) Is f commutative? (b) Is f associative? (c) Does f have an identity element?

a) For all $a, b \in \mathrm{R}$, $f(a, b) = \lceil a + b \rceil = \lceil b + a \rceil = f(b, a)$, because the real numbers are commutative under addition. Hence f is a commutative (closed) binary operation.

b) This binary operation is not associative. For example, $f(f(3.2, 4.7), 6.4) = f(\lceil 3.2 + 4.7 \rceil, 6.4) = f(\lceil 7.9 \rceil, 6.4)$ $= f(8, 6.4) = \lceil 8 + 6.4 \rceil = \lceil 14.4 \rceil = 15$, while, $f\big(3.2, f(4.7, 6.4)\big) = f(3.2, \lceil 4.7 + 6.4 \rceil) = f(3.2, \lceil 11.1 \rceil)$ $= f(3.2, 12) = \lceil 3.2 + 12 \rceil = \lceil 15.2 \rceil = 16$.

c) There is no identity element. If $a \in \mathrm{R} - Z$ then for any $b \in \mathrm{R}$, $\lceil a + b \rceil \in \mathrm{Z}$. So if x were the identity element we would have $a = f(a, x) = \lceil a + x \rceil$ with $a \in \mathrm{R} - Z$ and $\lceil a + x \rceil \in \mathrm{Z}$

Ex 5.4: (5)

5. Let $|A| = 5$. (a) What is $|A \times A|$? (b) How many functions $f: A \times A \to A$ are there? (c) How many closed binary operations are there on A? (d) How many of these closed binary operations are commutative?

a) 25

b) 5^{25}

c) 5^{25}

d) 5^{10}

Ex 5.4: (8)

8. Let $A = \{2, 4, 8, 16, 32\}$, and consider the closed binary operation $f: A \times A \to A$ where $f(a, b) = \gcd(a, b)$. Does f have an identity element?

- Each element in A is of the form 2^i for some $1 \leq i \leq 5$, and $\gcd(2^i, 2^5) = 2^i = \gcd(2^5, 2^i)$, so $2^5 = 32$ is the identity element for f.

Ex 5.4: (12)

12. Let $A = B = \mathbf{R}$. Determine $\pi_A(D)$ and $\pi_B(D)$ for each of the following sets $D \subseteq A \times B$.

a) $D = \{(x, y) | x = y^2\}$

b) $D = \{(x, y) | y = \sin x\}$

c) $D = \{(x, y) | x^2 + y^2 = 1\}$

a) $\pi_A(D) = [0, +\infty);\ \pi_B(D) = R$

b) $\pi_A(D) = R;\ \pi_B(D) = [-1,1]$

c) $\pi_A(D) = [-1,1];\ \pi_B(D) = [-1,1]$

Ex 5.5: (2)

2. Show that if eight people are in a room, at least two of them have birthdays that occur on the same day of the week.

- The result follows by the Pigeonhole Principle where the eight people are the pigeons and the pigeonholcs are the seven days of the week.

Ex 5.5: (6)

6. Prove that if we select 101 integers from the set $S = \{1, 2, 3, \ldots, 200\}$, there exist m, n in the selection where $\gcd(m, n) = 1$.

- Any selection of size 101 from S must contain two consecutive integers $n, n+1$ and $\gcd(n, n+1) = 1$.

Ex 5.5: (13)

13. Let S be a set of five positive integers the maximum of which is at most 9. Prove that the sums of the elements in all the nonempty subsets of S cannot all be distinct.

- Consider the subsets A of S where $1 \leq |A| \leq 3$. Since $|S| = 5$, there are $\binom{5}{1} + \binom{5}{2} + \binom{5}{3} = 25$ such subsets A. Let s_A denote the sum of the elements in A. Then $1 \leq s_A \leq 7 + 8 + 9 = 24$. So by the Pigeonhole Principle, there are two subsets of S whose elements yield the same sum.

Ex 5.5: (14)

14. During the first six weeks of his senior year in college, Brace sends out at least one resumé each day but no more than 60 resumés in total. Show that there is a period of consecutive days during which he sends out exactly 23 resumés.

- For $1 \leq i \leq 42$, let x_i count the total number of resumés Brace has sent out from the start of his senior year to the end of the i-th day. Then $1 \leq x_1 < x_2 < \cdots < x_{42} \leq 60$, and $x_1 + 23 < x_2 + 23 < \ldots < x_{42} + 23 \leq 83$. We have 42 distinct numbers $x_1, x_2, \ldots, x_{42}$, and 42 other distinct numbers $x_1 + 23, x_2 + 23, \ldots, x_{42} + 23$, all between 1 and 83 inclusive. By the Pigeonhole Principle $x_i = x_j + 23$ for some $1 \leq j < i \leq 42$; $x_i - x_j = 23$.

Ex 5.6: (7)

7. Let $f, g, h: \mathbf{Z} \to \mathbf{Z}$ be defined by $f(x) = x - 1$, $g(x) = 3x$,

$$h(x) = \begin{cases} 0, & x \text{ even} \\ 1, & x \text{ odd.} \end{cases}$$

Determine (a) $f \circ g$, $g \circ f$, $g \circ h$, $h \circ g$, $f \circ (g \circ h)$, $(f \circ g) \circ h$; (b) $f^2, f^3, g^2, g^3, h^2, h^3, h^{500}$.

a) $(f \circ g)(x) = 3x - 1; (g \circ f)(x) = 3(x - 1);$

$$(g \circ h)(x) = \begin{cases} 0, x\ even \\ 3, x\ odd \end{cases}; \ (h \circ g)(x) = \begin{cases} 0, x\ even \\ 1, x\ odd \end{cases}$$

$$\big(f \circ (g \circ h)\big)(x) = f\big((g \circ h)(x)\big) = \begin{cases} -1, x\ even \\ 2, x\ odd \end{cases}$$

$$\big((f \circ g) \circ h\big)(x) = \begin{cases} (f \circ g)(0), x\ even \\ (f \circ g)(1), x\ odd \end{cases} = \begin{cases} -1, x\ even \\ 2, x\ odd \end{cases}$$

b) $f^2(x) = f\big(f(x)\big) = x - 2; f^3(x) = x - 3;$

$g^2(x) = 9x; g^3(x) = 27x; h^2 = h^3 = h^{500} = h$

Ex 5.6: (10)

10. For each of the following functions $f : \mathbf{R} \to \mathbf{R}$, determine whether f is invertible, and, if so, determine f^{-1}.

a) $f = \{(x, y) | 2x + 3y = 7\}$

b) $f = \{(x, y) | ax + by = c, b \neq 0\}$

c) $f = \{(x, y) | y = x^3\}$

d) $f = \{(x, y) | y = x^4 + x\}$

a) $f^{-1} = \{(x, y) | 2y + 3x = 7\}$

b) $f^{-1} = \{(x, y) | ay + bx = c, b \neq 0, a \neq 0\}$

c) $f^{-1} = \left\{(x, y) \middle| y = x^{\frac{1}{3}}\right\} = \{(x, y) | x = y^3\}$

d) Here $f(0) = f(-1) = 0$, so f is not one-to-one, and consequently f is not invertible.

Ex 5.6: (16)

16. Let $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = \lfloor x \rfloor$, the greatest integer in x. Find $f^{-1}(B)$ for each of the following subsets B of $\mathbf{R}$.

a) $B = \{0, 1\}$
b) $B = \{-1, 0, 1\}$
c) $B = [0, 1)$
d) $B = [0, 2)$
e) $B = [-1, 2]$
f) $B = [-1, 0) \cup (1, 3]$

a) $[0,2)$

b) $[-1,2)$

c) $[0,1)$

d) $[0,2)$

e) $[-1,3)$

f) $[-1,0) \cup [2,4)$

Ex 5.6: (17)

17. Let $f, g: \mathbf{Z}^+ \to \mathbf{Z}^+$ where for all $x \in \mathbf{Z}^+$, $f(x) = x + 1$ and $g(x) = \max\{1, x - 1\}$, the maximum of 1 and $x - 1$.

a) What is the range of f?

b) Is f an onto function?

c) Is the function f one-to-one?

d) What is the range of g?

e) Is g an onto function?

f) Is the function g one-to-one?

g) Show that $g \circ f = 1_{\mathbf{Z}^+}$.

h) Determine $(f \circ g)(x)$ for $x = 2, 3, 4, 7, 12$, and 25.

i) Do the answers for parts (b), (g), and (h) contradict the result in Theorem 8?

Ex 5.6: (17.a~17.g)

a) The range of $f = \{2,3,4,\ldots\} = Z^+ - \{1\}$.

b) Since 1 is not in the range of f. The function is not onto.

c) For all $x, y \in \mathrm{Z}^+, f(x) = f(y) \Rightarrow x + 1 = y + 1 \Rightarrow x = y$, so f is one-to-one.

d) The range of g is Z^+.

e) Since $g(Z^+) = Z^+$, the codomain of g, this function is onto.

f) Here $g(1) = 1 = g(2)$, and $1 \neq 2$, so g is not one-to-one.

g) For all $x \in \mathrm{Z}^+, (g \circ \mathrm{f})(x) = g\big(f(x)\big) = g(x+1) = \max\{1, (x+1) - 1\} = \max\{1, x\} = x, since\ x \in \mathrm{Z}^+$. Hence $g \circ \mathrm{f} = 1_{Z^+}$.

Ex 5.6: (17.h & 17.i)

h) $(f \circ g)(2) = f(\max\{1,1\}) = f(1) = 1 + 1 = 2$
$(f \circ g)(3) = f(\max\{1,2\}) = f(2) = 2 + 1 = 3$
$(f \circ g)(4) = f(\max\{1,3\}) = f(3) = 3 + 1 = 4$
$(f \circ g)(7) = f(\max\{1,6\}) = f(6) = 6 + 1 = 7$
$(f \circ g)(12) = f(\max\{1,11\}) = f(11) = 11 + 1 = 12$
$(f \circ g)(25) = f(\max\{1,24\}) = f(24) = 24 + 1 = 25$

i) No, because the functions f, g are not inverses of each other. The calculations in part (h) may suggest that $f \circ g = 1_{Z^+}$ since $(f \circ g)(x) = x \; for \; x \geq 2$. But we also find that $(f \circ g)(1) = f(\max\{1,0\}) = f(1) = 2$, so $(f \circ g)(1) \neq 1$, and, consequently, $f \circ g \neq 1_{Z^+}$.

Ex 5.6: (22)

22. If $|A| = |B| = 5$, how many functions $f: A \to B$ are invertible?

- It follows from Theorem 5.11 that there are 5! Invertible functions $f: A \to B$.

EX 5.5: (20)

- How many times must we roll a single die in order to get the same score (a) at least twice (b) at least three times (c) at least n times, for $n \geq 4$
- (a) 7
- (b) 13
- (c) $6(n-1)+1$