Name: Student ID:

Quiz #5 (5%)

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

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3:30 - 3:50 p.m., April 21st, 2014

This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (1%) If $A = \{1, 2, 3, 4, 5\}$ and there are 2520 injective functions $f : A \to B$, what is $|B|$?

   Solution:

   The number of injective functions is $\frac{|B|!}{(|B|-5)!} = 2520$.

   Therefore, $|B| = 7$.

2) (2%) Answer the following questions.

   a) How many ways can 31,100,905 be factored into three factors, each greater than 1, if the order of the factors is irrelevant?

   b) Answer part (a), assuming the order of the three factors is relevant.

   Solution:

   $31100905 = 5 \times 11 \times 17 \times 29 \times 31 \times 37$

   a) Consider the problem as to distribute those 6 prime factors $\{5, 11, 17, 29, 31, 37\}$ into 3 identical containers with no container left empty, then there are $S(6, 3) = 90$ ways.

   b) If the order is considered, that is, the containers are different.
      Then, there are $S(6, 3) \times 3! = 540$ ways.

3) (2%) Let $|A| = 5$, answer the following questions

   a) What is $|A \times A|$?

   b) How many functions $f : A \times A \to A$ are there?

   c) How many closed binary operations are there on A ?

d) How many of these closed binary operations are commutative?

Solution:

a) $5 \times 5 = 25$

b) 5^{25}

c) 5^{25}

d) **Because the solution manual gave a wrong answer for this question, you all get the credits.**

   f is said to be commutative if $f(x, y) = f(y, x)$ for all $(x, y) \in A \times A$.

   Consider the following 2 conditions:

   (1) $x = y$. Then, there 5 of them.

   (2) $x \neq y$. Then, there are $25 - 5 = 20$ of them. We need to ensure that $f(x, y) = f(y, x)$, and there $20/2 = 10$ sets of these kinds two ordered pairs.

   Therefore, the number of commutative closed binary operations f on A is $5^{5+10} = 5^{15}$.