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Quiz #7 (5% + 1% Bonus Point)

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

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2:20 - 2:40 p.m., May 14th, 2014

This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (2%) For $A = \{a, b, c, d, e, f\}$, each graph, or digraph, in the figure, represents a relation $\mathscr{R}$ on A. Write each relation $\mathscr{R} \subseteq A \times A$ as a relation matrix $M(\mathscr{R})$.

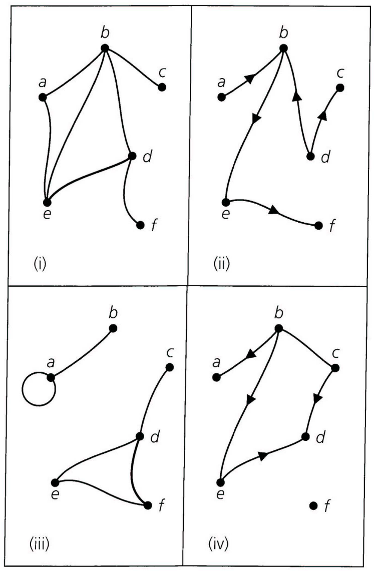


Figure 7.13

Solution:

a) $\mathscr{R} = (a, b), (b, a), (a, e), (e, a), (b, c), (c, b), (b, d), (d, b), (b, e), (e, b), (d, e), (e, d), (d, f), (f, d)$

$$M(\mathscr{R}) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

b) $\mathscr{R} = (a,b),(b,e),(d,b),(d,c),(e,f)$

$$M(\mathscr{R}) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

c) $\mathscr{R} = (a,a),(a,b),(b,a),(c,d),(d,c)(d,e),(e,d),(d,f),(f,d),(e,f),(f,e)$

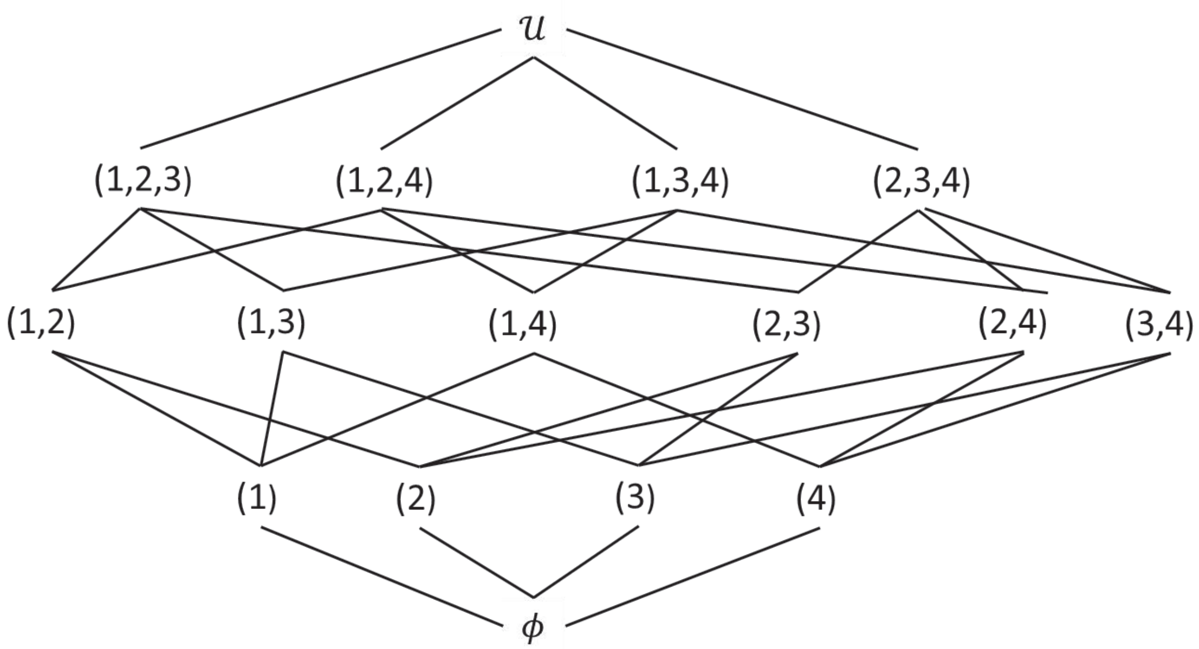
$$M(\mathscr{R}) = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

d) $\mathscr{R} = (b,a),(b,c),(c,b)(b,e),(c,d),(e,d)$

$$M(\mathscr{R}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2) (2%) Draw the Hasse diagram for the poset $(\mathscr{P}(\mathscr{U}), \subseteq)$, where $\mathscr{U} = \{1, 2, 3, 4\}$.

Solution:



3) (2%) Apply the minimization process to each finite state machine in the figure.

Solution:

a) $P_1 : \{s_1, s_4\}, \{s_2, s_3, s_5\}$

$(\nu(s_1, 0) = s_4)E_1(\nu(s_4, 0) = s_1$ but $(\nu(s_1, 1) = s_1)\neg E_1(\nu(s_4, 1) = s_3)$, so $s_1 \neg E_2 s_4$

$(\nu(s_2, 1) = s_3)E_1(\nu(s_3, 1) = s_4$, so $s_2 \neg E_2 s_3$

$(\nu(s_2, 0) = s_3)E_1(\nu(s_5, 0) = s_3$, and $(\nu(s_2, 1) = s_3)E_1(\nu(s_5, 1) = s_3$, so $s_2 E_2 s_5$

Since $s_2 \neg E_2 s_3$ and $s_2 E_2 s_5$. It follows that $s_2 \neg E_2 s_5$

Hence, P_2 is given by $P_2 : \{s_1\}, \{s_2, s_5\}, \{s_3\}, \{s_4\}$

$(\nu(s_2, x) = s_3)E_2(\nu(s_5, x) = s_3$ for $x = 0, 1$. Hence, $s_2 E_2 s_5$ and $P_2 = P_3$

b) State s_2 and s_5 are equivalent

	ν		ω	
	0	1	0	1
s_1	s_4	s_1	0	1
s_2	s_3	s_3	1	0
s_3	s_1	s_4	1	0
s_4	s_1	s_3	0	1
s_5	s_3	s_3	1	0

(a)

	ν		ω	
	0	1	0	1
s_1	s_6	s_3	0	0
s_2	s_5	s_4	0	1
s_3	s_6	s_2	1	1
s_4	s_4	s_3	1	0
s_5	s_2	s_4	0	1
s_6	s_4	s_6	0	0

(b)