Name: Student ID:

Quiz #9 (4% + 1% Bonus Point)

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

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2:20 - 2:40 p.m., June 5th, 2014

1) (1%) Find the generating function for the number of ways to select 10 candy bars from large supplies of 6 different kinds.

Solution:

$(1 + x + x^2 + x^3 ... + x^{10})^6$

2) (2%) Find the coefficient of x^{15} in each of the following: (a) $x^3(1 - 2x)^{10}$ and (b) $(1 + x)^4/(1 - x)^4$.

Solution:

a) 0

b) $\binom{4}{0}\binom{-4}{15}(-1)^{15} + \binom{4}{1}\binom{-4}{14}(-1)^{14} + \binom{4}{2}\binom{-4}{13}(-1)^{13} + \binom{4}{3}\binom{-4}{12}(-1)^{12} + \binom{4}{4}\binom{-4}{11}(-1)^{11} = \binom{4}{0}\binom{18}{15} + \binom{4}{1}\binom{17}{14} + \binom{4}{2}\binom{16}{13} + \binom{4}{3}\binom{15}{12} + \binom{4}{4}\binom{14}{11}$

3) (2%) Find the generating function for the number of integer solutions of: (a) $2w + 3x + 5y + 7z = n$, $0 \leq w, x, y, z$ and (b) $2w + 3x + 5y + 7z = n$, $0 \leq w$, $4 \leq x, y$, $5 \leq z$.

Solution:

a) $\frac{1}{1-x^2} \cdot \frac{1}{1-x^3} \cdot \frac{1}{1-x^5} \cdot \frac{1}{1-x^7}$

b) $\frac{1}{1-x^2} \cdot \frac{x^{12}}{1-x^3} \cdot \frac{x^{20}}{1-x^5} \cdot \frac{x^{35}}{1-x^7}$