Name: Student ID:

Quiz #1 6%

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

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This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (1%) In how many ways can the symbols a, b, b, c, d, e, e, e, e, e, e be arranged so that no e is adjacent to another e?

   Answer: There are $\frac{5!}{2!} = 60$ *ways.*

2) *(1%) Consider the following program segment where i, j, and k are integer variables.*

```
for i := 1 to 12 do
    for j := 5 to 10 do
        read (i-j)
        for k := 15 downto 8 do
            print (i-j)*k
```

   a) *How many time is the* **print** *statement executed?*
   b) *How many time is the* **read** *statement executed?*

   Answer:

   a) The line will be executed $(12 - 1 + 1) \times (10 - 5 + 1) \times (15 - 8 + 1) = 576$ times.
   b) The line will be executed $(12 - 1 + 1) \times (10 - 5 + 1) = 72$ times.

3) (2%) Determine the coefficient of $w^2x^2y^2z^2$ in the expansion of

a) $(w-x+y-z)^8$

b) $(w+x+y+z+1)^2$

c) $(2w-x+3y+z-2)^{12}$

d) $(v+w-2x+y+5z+3)^{12}$

Answer:

a) $\binom{8}{2,2,2,2}(1)^2(-1)^2(1)^2(-1)^2 = \frac{8!}{2!^4} \times 1$

b) 0

c) $\binom{12}{2,2,2,2,4}(2)^2(-1)^2(3)^2(1)^2(-2)^4 = \frac{12!}{2!^4\,4!} \times (2)^2(3)^2(2)^4$

d) $\binom{12}{0,2,2,2,2,4}(1)^2(-2)^2(1)^2(5)^2(3)^4 = \frac{12!}{2!^4\,4!} \times (2)^2(5)^2(3)^4$

4) *(2%) Consider the strings made up of n bits – that is, a total of n 0's and 1's. In particular consider those strings with exactly five occurrences of* 01*. For* $n \geq 10$*, How many such strings are there?*

Answer:

For $n \geq 10$, a string with this structure has x_1 1's followed by x_2 0's followed by x_3 1's ... followed by x_{12} 0's, where $x_1 + x_2 + ... + x_{12} = n$, $x_1, x_{12} \geq 0$, $x_2, ..., x_{11} > 0$.

The number of solutions to this equation equals to the number of solutions to $y_1 + y_2 + ... + y_{12} = n - 10$. The number of this equation is $\binom{12+(n-10)-1}{n-10} = \binom{n+1}{11}$