Name: Student ID:

Quiz #3 6%

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This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (2%) For $A = \{1, 2, 3, 4, 5, 6, 7\}$, determine the number of
   a) subsets of A
   b) nonempty proper subsets of A
   c) subsets of A containing three elements
   d) subsets of A containing five elements, including 1 and 2

   Answer:
   a) 2^7
   b) 126
   c) $\binom{7}{3}$
   d) $\binom{5}{3}$

2) *(2%) The freshman class of a private engineering college has 300 students. It is known that* 180 *can program in Java,* 120 *in Visual BASIC,* 30 *in C++,* 12 *in Java and C++,* 18 *in Visual BASIC and C++,* 12 *in Java and Visual BASIC, and* 6 *in all three languages. A student is selected at random.*
   a) *What is the probability that he/she can program in exactly two languages?*
   b) *What is the probability that he/she can program in none of the languages?*

Answer:

a) *The number of students that can program in exactly two languages is* $(12-6)+(18-6)+(12-6)=24$. *So the probability is* $\frac{24}{300}=0.08$.

b) *The number of students that can program in none of the language is total number of the students minus number of students that know at least one language. Which is* $180+120+30-12-18-12+6=294$. *So the probability is* $\frac{300-294}{300}=\frac{6}{300}=0.02$.

3) *(2%) Using the laws of set theory, simplify each of the following:*

a) $A \cap (B-A)$

b) $\overline{A} \cup \overline{B} \cup (A \cap B \cap \overline{C})$

Answer:

a) $A \cap (B-A)$

$= A \cap (B \cap \overline{A})$ *reason: Definition of relative complement*

$= B \cap (A \cap \overline{A})$ *reason: Associative Laws*

$= B \cap \emptyset$ *reason: Inverse Laws*

$= \emptyset$ *reason: Domination Laws*

b) $\overline{A} \cup \overline{B} \cup (A \cap B \cap \overline{C})$

$= \overline{(A \cap B)} \cup [(A \cap B) \cap \overline{C}]$ *reason: DeMorgan's Laws*

$= [\overline{(A \cap B)} \cup (A \cap B)] \cap [\overline{(A \cap B)} \cup \overline{C}]$ *reason: Distributive Laws*

$= \mathbb{U} \cap (\overline{A \cap B} \cup \overline{C})$ *reason: Inverse Laws*

$= \overline{A \cap B} \cup \overline{C}$ *reason: Identity Laws*

$= \overline{A} \cup \overline{B} \cup \overline{C}$ *reason: DeMorgan's Laws*