

Name:

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Quiz #5 6%

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This is a closed book test. Any academic dishonesty will automatically lead to zero point.

- 1) (1%) A rumor is spread as follows. The originator calls two people. Each of these people phones three friends, each of whom in turn calls five associates. If no one receives more than one call, and no one calls the originator.
- How many people now know the rumor?
 - How many phone calls were made?

Answer:

a) $1 + 2 + 2*3 + 2*3*5 = 39$

b) $2 + 2*3 + 2*3*5 = 38$

- 2) (1%) If 11 integers are selected from $\{1, 2, 3, \dots, 100\}$, prove that there are at least two, say x and y , such that $0 < |\sqrt{x} - \sqrt{y}| < 1$.

Answer:

For any $t \in \{1, 2, 3, \dots, 100\}$, $1 \leq \sqrt{t} \leq 10$, i.e. there are only 10 different possible values for \sqrt{t} . Here we take possible value as pigeonholes, and the number of element as pigeons. By the Pigeonhole Principle, selecting 11 elements from $\{1, 2, 3, \dots, 100\}$ there must be two, say x and y , where $\lfloor \sqrt{x} \rfloor = \lfloor \sqrt{y} \rfloor$, so that $0 < |\sqrt{x} - \sqrt{y}| < 1$.

3) (2%) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$. For each of the following subsets B of \mathbb{R} , find $f^{-1}(B)$.

- a) $B = [0, 4]$
- b) $B = \{-1, 0, 1\}$
- c) $B = [0, 1)$
- d) $B = (0, 1] \cup (4, 9)$

Answer:

- a) $[-2, 2]$
- b) $\{-1, 0, 1\}$
- c) $(-1, 1)$
- d) $(-3, -2) \cup [-1, 0) \cup (0, 1] \cup (2, 3)$

4) (2%) For each of the following functions $f: \mathbb{Z} \rightarrow \mathbb{Z}$, determine whether the function is one-to-one and whether it is onto. If the function is not onto, determine the range $f(\mathbb{Z})$.

- a) $f(x) = x^2$
- b) $f(x) = x + 7$
- c) $f(x) = 2x - 3$
- d) $f(x) = x^3$

Answer:

- a) Since $f(-1) = f(1)$, f is not one-to-one. Also f is not onto. The range of $f = \{0, 1, 4, 9, 16, \dots, n^2\}, n \in \mathbb{N}$.
- b) One-to-one and onto.
- c) One-to-one but not onto. The range consist of all the odd integers.
- d) One-to-one but not onto. The range of $f = \{\dots, -64, -27, -8, -1, 0, 1, 8, 27, \dots\}$.