Name: Student ID:

Quiz #8 6%

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

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This is a closed book test. Any academic dishonesty will automatically lead to zero point. If the total points are more than 6 points, you will get at most 6 points out of it.

1) (2%) Five professors named Al, Violet, Lynn, Jack and Mary Lou are to be assigned to teach one class each from among calculus I, calculus II, calculus III, statistics, and combinatorics. Al will not teach calculus II or combinatorics, Lynn cannot stand statistics, Violet and Mary Lou both refuse to teach calculus I or calculus III, and Jack detests calculus II. (No points if none of the intermediate steps is given)
   a) In how many ways can the head of the mathematics department assign each of these professors one of these five courses and still keep peace in the department?
   b) For the assignments in part (a), what is the probability that Violet will get to teach combinatorics?

   Answer:

   a) Let $t_i, 1 \leq i \leq 5$, denote the following conditions.
      - t_1: Al have to teach calculus II (C_2 in the chessboard) or combinatorics (C in the chessboard).
      - t_2: Lynn have to teach statistics (S in the chessboard).
      - t_3: Violet have to teach calculus I (C_1 in the chessboard) or calculus III (C_3 in the chessboard).
      - t_4: Mary Lou have to teach calculus I or calculus III.
      - t_5: Jack have to teach calculus II.

      Here we construct the chessboard B_1 and calculate the rook polynomial
      $r(B_1, x) = (1 + 4x + 2x^2)(1 + 3x + x^2)(1 + x)$

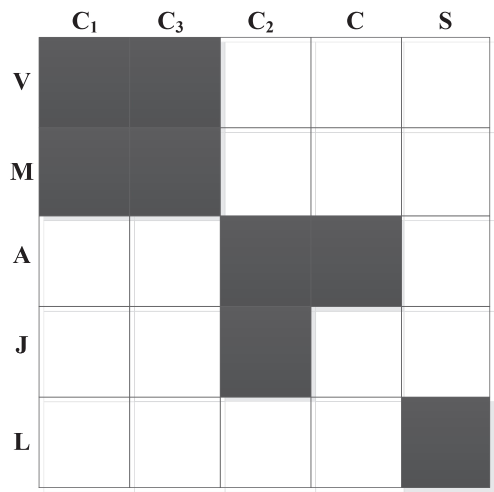
$= 1 + 8x + 22x^2 + 25x^3 + 14x^4 + 2x^5$

Therefore,

$N(\overline{t_1}\ \overline{t_2}\ \overline{t_3}\ \overline{t_4}\ \overline{t_5})$

$= 5! - 8(4!) + 22(3!) - 25(2!) + 12(1!) - 2(0!)$

$= 20$



b) In this problem we need to modify t_3 in the previous problem, so that it become t_3': Violet have to teach calculus I (C_1 in the chessboard) or calculus III (C_3 in the chessboard) or combinatorics.

Then we reconstruct the chessboard B_2 and calculate the rook polynomial

$r(B_2, x) = (1 + 8x + 19x^2 + 14x^3 + 2x^4)(1 + x)$

$= 1 + 9x + 27x^2 + 33x^3 + 16x^4 + 2x^5$

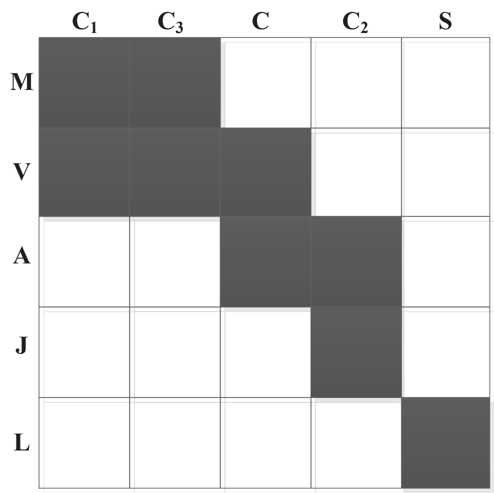
Therefore,

$N(\overline{t_1}\ \overline{t_2}\ \overline{t_3'}\ \overline{t_4}\ \overline{t_5})$

$= 5! - 9(4!) + 27(3!) - 33(2!) + 16(1!) - 2(0!)$

$= 14$

These are the number of ways in (a) which Violet doesn't get to teach combinatorics. So the final answer is $(20 - 14)/20 = 3/10$.



2) (2%) How many derangements are there for $1, 2, 3, 4, 5$?

Answer:

$5![1 - 1 + (\frac{1}{2!}) - (\frac{1}{3!}) + (\frac{1}{4!}) - (\frac{1}{5!})]$
$= (5)(4)(3) - (5)(4) + 5 - 1 = 60 - 20 + 5 - 1 = 44$

3) (2%) Find the number of permutations of $a, b, c, ..., x, y, z$, in which none of the patterns *spin*, *game*, *path*, or *net* occurs.

Answer:

Let c_1 denote that the arrangement contains the pattern *spin*. Likewise, let c_2, c_3, c_4 denote this for the patterns *game*, *path* and *net*, respectively. $N(\overline{c_1}\ \overline{c_2}\ \overline{c_3}\ \overline{c_4}) = 26! - [3(23!) + 24!] + (20! + 21!)$

4) (2%) Zelma is having a luncheon for herself and nine of the women in her tennis league. On the morning of the luncheon she places name cards at the ten places at her table and then leaves to run a last-minutes errand. Her husband, Herbert, comes home from his morning tennis match and unfortunately leaves the back door open. A gust of wind scatters the ten name cards. In how many way can Herbert replace the ten cards at the places at the table so that

   a) Exactly four of the ten women will be seated where Zelma had wanted them?

b) At least four of the ten women will be seated where Zelma had wanted them? (Please use S_i, where $S_i = \binom{10}{i}(i!)$, for $1 \leq i \leq 10$, to answer the problem.)

Answer:

a) $E_4 = S_4 - \binom{5}{1}S_5 + \binom{6}{2}S_6 - \binom{7}{3}S_7 + \binom{8}{4}S_8 - \binom{9}{5}S_9 + \binom{10}{6}S_{10}$

b) $L_4 = S_4 - \binom{4}{3}S_5 + \binom{5}{3}S_6 - \binom{6}{3}S_7 + \binom{7}{3}S_8 - \binom{8}{3}S_9 + \binom{9}{3}S_{10}$