Name: Student ID:

Quiz #9 6%

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

Department of Computing Science, National Tsing Hua University, Taiwan

This is a closed book test. Any academic dishonesty will automatically lead to zero point. If the total points are more than 6 points, you will get at most 6 points out of it.

1) (3%) Determine the generating function for the number of ways to distribute 35¢ (from an unlimited supply) among five children if
   a) there are no restrictions.
   b) each child gets at least 1¢.
   c) each child gets at least 2¢.
   d) the oldest child gets at least 10¢.
   e) the two youngest children must each get at least 10¢.
   f) the oldest child gets more than 3¢, but no more than 20¢.

Answer:

a) $(1+x+x^2+\cdots+x^{35})^5$ or $(1+x+x^2+\ldots)^5$
b) $(x+x^2+\cdots+x^{35})^5$ or $x^5(1+x+x^2+\ldots)^5$
c) $(x^2+x^3+\cdots+x^{35})^5$ or $x^{10}(1+x+x^2+\ldots)^5$
d) $(1+x+x^2+x^3+\cdots+x^{25})^4(x^{10}+x^{11}+\cdots+x^{35})$ or $(1+x+x^2+\ldots)^4(x^{10}+x^{11}+x^{12}+\ldots)$
e) $(x^{10}+x^{11}+\cdots+x^{25})^2(1+x+x^2+x^3+\cdots+x^{15})^3$ or $(x^{10}+x^{11}+\ldots)^2(1+x+x^2+\ldots)^3$
f) $(x^4+x^5+\cdots+x^{20})(1+x+x^3+\cdots+x^{31})^4$ or $(x^4+x^5+\ldots)(1+x+x^3+\ldots)^4$

2) (2%) Determine the sequence generated by each of the following exponential generating functions.

   a) $f(x) = e^x + e^{-x}$
   b) $f(x) = 7e^{7x} - 3e^{2x}$

Answer:

   a) $2, 0, 2, 0, 2, 0, \ldots$.
   b) $f(x) = 7e^{7x} - 3e^{2x} = 7\sum_{i=0}^{\infty} \frac{(7x)^i}{i!} - 3\sum_{i=0}^{\infty} \frac{(2x)^i}{i!}$, so $f(x)$ is exponential generating function for the sequence $4, 43, 331, \ldots, 7(7^n) - 3(2^n), \ldots$.

3) (2%) Determine the constant (that is, the coefficient of x^0) in $(3x^2 - (2/x))^{15}$.

Answer:

$\binom{15}{5}(3^5)(2^{10})$

4) (2%) In how many ways can two dozen identical robots be assigned to four assembly lines with

   a) at least three robots assigned to each line?
   b) at least three, but no more than nine robots assigned to each line?

(You don't need to calculate the actual result of combination in the answer, but please give an answer without negative number in the combination)

Answer:

   a) $(x^3 + x^4 + \ldots)^4 = x^{12}(1 + x + x^2 + \ldots)^4 = x^{12}(1 - x)^{-4}$. The coefficient of x^{12} in $(1 - x)^{-4}$ is $\binom{-4}{12}(-1)^{12} = (-1)^{12}\binom{4+12-1}{12}(-1)^{12} = \binom{15}{12}$.
   b) $(x^3 + x^4 + \cdots + x^9)^4 = x^{12}(1 + x + x^2 + \cdots + x^6)^4$. The coefficient of x^{12} in $[(1-x)^7/(1-x)]^4 = (1-x^7)^4(1-x)^{-4} = [1-4x^7+\cdots+x^{28}][\binom{-4}{0}+\cdots+\binom{-4}{5}(-x)^5 + \cdots + \binom{-4}{12}(-x)^{12} + \ldots]$ is $(-4)\binom{-4}{5}(-1)^5 + \binom{-4}{12}(-1)^{12} = (4)(-1)^5\binom{8}{5} + \binom{15}{12} = \binom{15}{12} - 4\binom{8}{5}$.