SOLUTION

- Exercise 6.1: 1, 3, 4, 7, 14
- Exercise 6.2: 3, 5, 6, 8, 9
- Exercise 6.3: 1, 2, 5, 7

Ex. 6.1-1

a) 25; 125

b) $\sum_{i=0}^{5} 5^i = 3906$

Ex. 6.1-3

- 12

Ex. 6.1-4

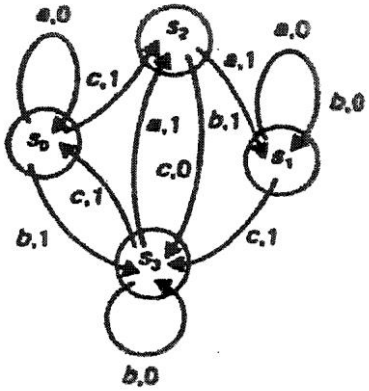
a) 0
b) 0
c) 1
d) 2
e) 3
f) 4
g) 1
h) 0

Ex. 6.1-7

a) $\{00,11,000,111,0000,1111\}$

b) $\{0,1\}$

c) $\sum^{*} - \{\lambda,00,11,000,111,0000,1111\}$



d) $\{0,1,00,11\}$

e) $\sum^{*}$

f) $\sum^{*} - \{0,1,00,11\} = \{\lambda,01,10\} \cup \{w \mid \| w \| \geq 3\}$

Ex. 6.1-14

There are five possible choices

(1) $A = \{\lambda\},\ \ B = \{01,000,0101,0111,01000,010111\};$

(2) $A = \{01,000,0101,0111,01000,010111\},\ \ B = \{\lambda\};$

(3) $A = \{0\},\ \ B = \{1,00,101,111,1000,10111\};$

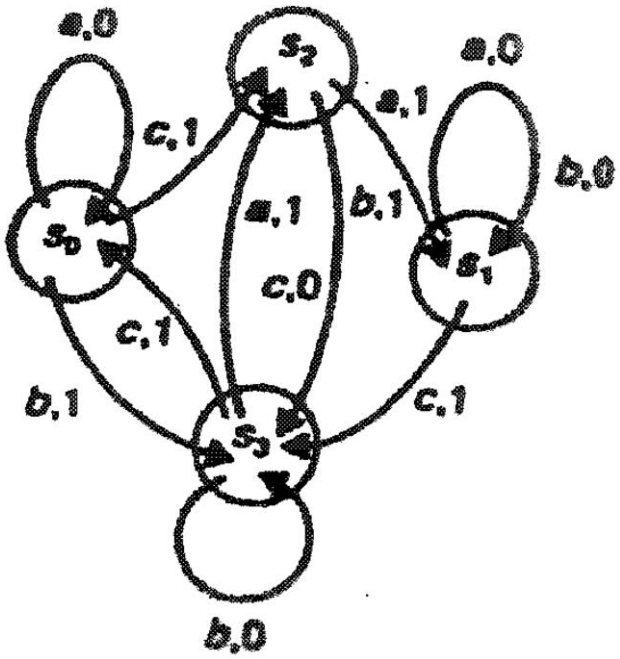
(4) $A = \{0,010\},\ \ B = \{1,00,111\};$

(5) $A = \{\lambda,01\},\ \ B = \{01,000,0111\};$

Ex. 6.2-3

a) 010110

b)



Ex. 6.2-5

a) 010000; s_2

b) (s_1)100000; s_2 (s_2)000000; s_2 (s_3) 110010; s_2

c)

	v		w	
	0	1	0	1
s_0	s_0	s_1	0	0
s_1	s_1	s_2	1	1
s_2	s_2	s_2	0	0
s_3	s_0	s_3	0	1
s_4	s_2	s_3	0	1

d) s_1

e) x = 101 (unique)

Ex. 6.2-6

a) The machine recognizes (with an output of 1) every 0 (in an input string x) that is proceed by another 0

b) State s_1 remembers that at least one 0 has been supplied from an input string x

c) A={1}*, b={00}

Ex. 6.2-8

a)

Input	0	1	1	0	1	1	1	0	1	1
Output	0	0	0	0	0	0	0	0	1	0

b)

	v		w	
	0	1	0	0
s_0	s_0	s_1	0	0
s_1	s_1	s_2	0	0
s_2	s_2	s_3	0	0
s_3	s_3	s_4	0	0
s_4	s_4	s_5	0	0
s_5	s_5	s_0	0	1

Ex. 6.2-8

c) $w(x, s_0) = 0000001$ for x =
   1) 1111101
   2) 1111011
   3) 1110111
   4) 1101111
   5) 1011111
   6) 0111111

d) The machine recognizes the occurrence of a sixth 1, a 12^{th} 1 in an input x

Ex. 6.2-9

a)

	v		w	
	0	1	0	0
s_0	s_4	s_1	0	0
s_1	s_3	s_2	0	0
s_2	s_3	s_2	0	1
s_3	s_3	s_3	0	0
s_4	s_5	s_3	0	0
s_5	s_5	s_3	1	0

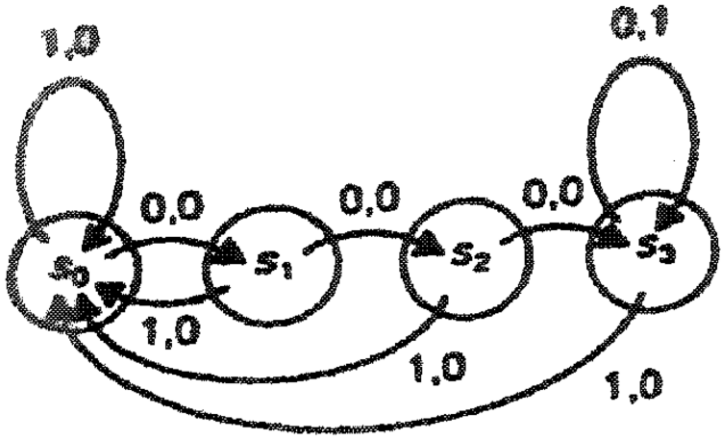
a) There are only two possibilities : x =1111 or x =0000

b) $A = \{111\}\{1\}^* \cup \{000\}\{0\}^*$

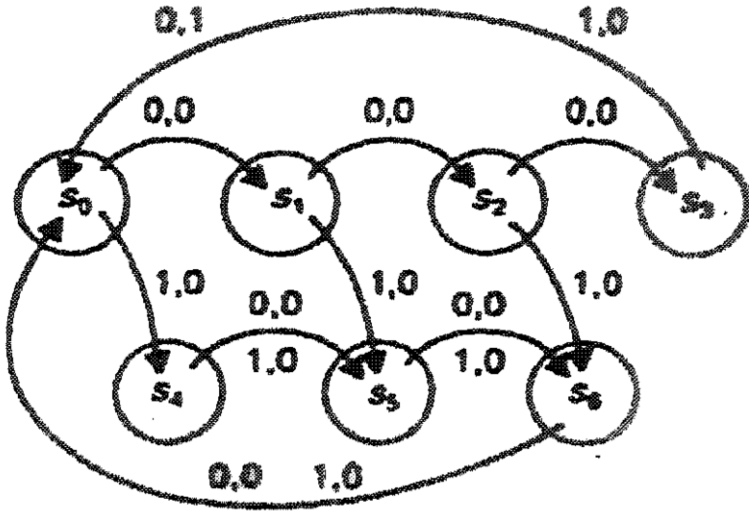
c) $A = \{11111\}\{1\}^* \cup \{00000\}\{0\}^*$

Ex. 6.3-1

a)

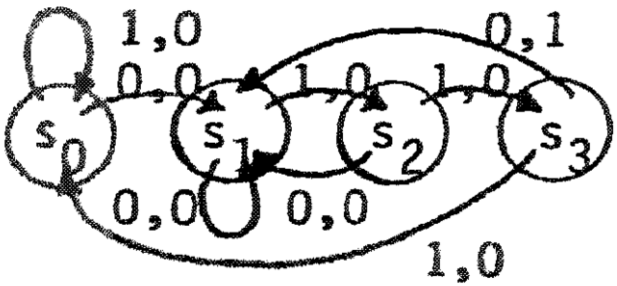

1,0
0,1
0,0
0,0
0,0
S_0
S_1
S_2
S_3
1,0
1,0
1,0

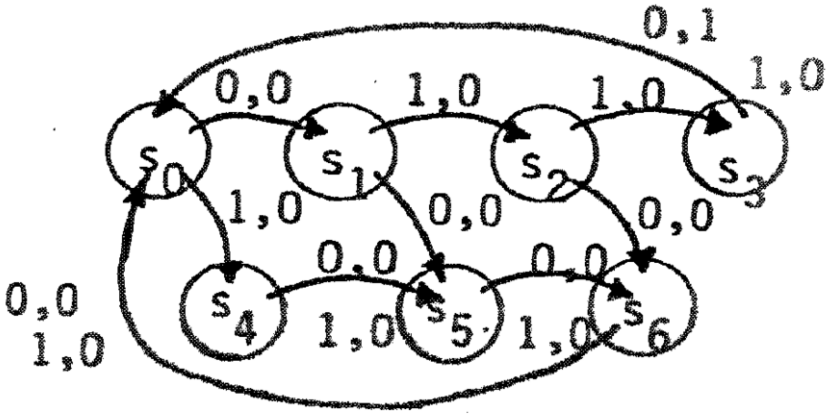
b)


0,1
1,0
0,0
0,0
0,0
S_0
S_1
S_2
S_3
1,0
0,0
1,0
0,0
1,0
1,0
1,0
S_4
S_5
S_6
0,0
1,0

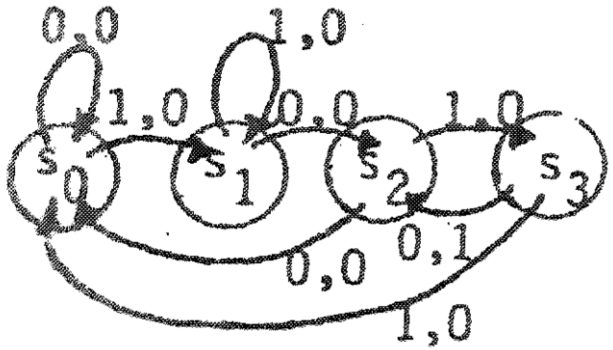
Ex. 6.3-2

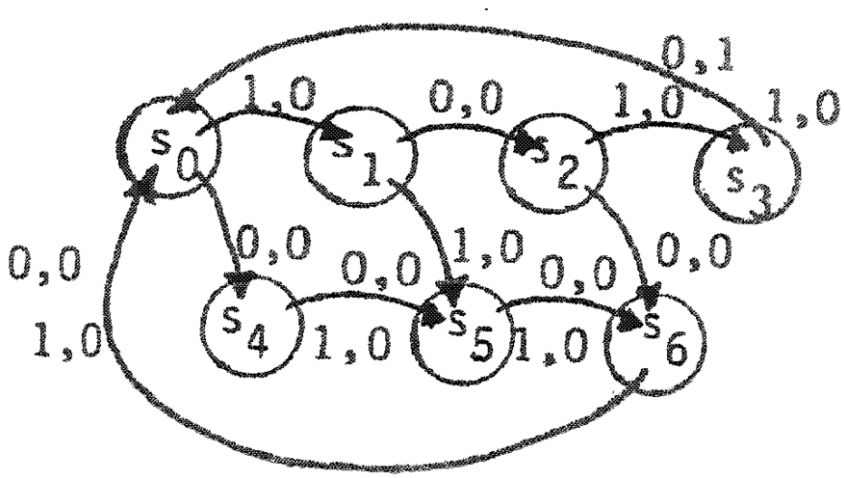
a) (0110)





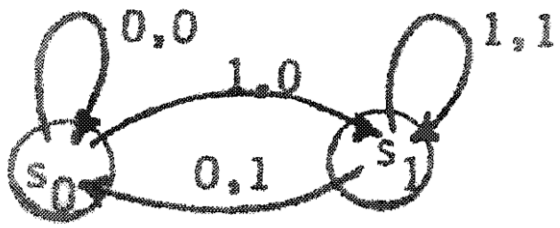
b) (1010)





Ex. 6.3-5

a)



b)

1) Input 111, output 011
2) Input 1010, output 0101
3) Input 00011, output 00001

c) The machine outputs a 0 followed by the first n-1 symbols of the n symbol input string x. Hence the machine is a unit delay.

d) The machine here performs the same tasks as the one in Fig. 6.13 and has only two states

Ex. 6.3-7

a) The transient states are s_0 , s_1. State s_4 is a sink state. $\{s_1, s_2, s_3, s_4, s_5\}$,$\{s_4\}$,$\{s_2 ,s_3 ,s_5\}$ (with the corresponding restrictions on the given function v) constitute submachines. The strongly connected submachines are $\{s_4\}$,$\{s_2 ,s_3 ,s_5\}$

b) States s_2 , s_3 are transient. The only sink state is s_4. The set $\{s_0, s_1 , s_3 , s_4\}$ provides the states for a submachine; $\{s_4\}$,$\{s_0,s_1\}$ provide strongly connected submachines

c) Here is no transient state. State s_6 is a sink state. There are three submachines: $\{s_2, s_3, s_4, s_5, s_6\}$,$\{s_6\}$, and $\{s_3 ,s_4 ,s_5 , s_6\}$. The strongly connected submachine is $\{s_6\}$