Name: Student ID:

Quiz #10 (4% + 2% Bonus)

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

Department of Computing Science, National Tsing Hua University, Taiwan

5:00 - 5:20 p.m., June 3rd, 2013

This is a closed book test. Any academic dishonesty will automatically lead to zero point.

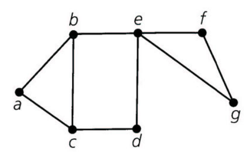
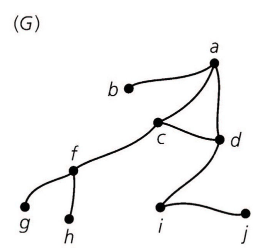


Figure 11.7

1) (1%) How many path are there from b to g?

   Answer:

   6



2) (1%) (a) How many spanning subgraphs are there for the graph G? (b) how many of them are connected subgraphs?

   Answer:

   a) $2^9 = 512$

   b) 4

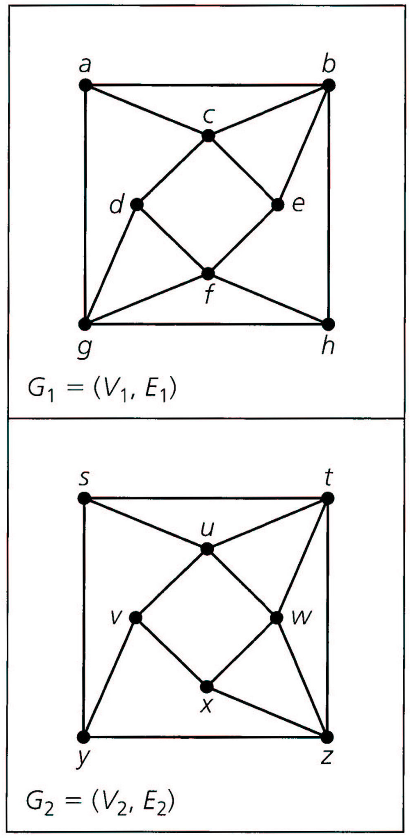


Figure 11.42

3) (1%) Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be loop-free undirected connected graphs in the figure. (a) Determine $|V_1|$, $|E_1|$, $|V_2|$, and $|E_2|$. (b) Find the degree of each vertex in V_1 and V_2. (c) Are G_1 and G_2 isomorphic?

Answer:

a) $|V_1| = 8, |E_1| = 14.|V_2| = 8, |E_2| = 14.$

b) $V_1 \Rightarrow \begin{cases} deg(a) = 3 \quad deg(b) = 4 \quad deg(c) = 4 \quad deg(d) = 3 \\ deg(e) = 3 \quad deg(f) = 4 \quad deg(g) = 4 \quad deg(h) = 3 \end{cases}$

$V_2 \Rightarrow \begin{cases} deg(s) = 3 \quad deg(t) = 4 \quad deg(u) = 4 \quad deg(v) = 3 \\ deg(w) = 4 \quad deg(x) = 3 \quad deg(y) = 4 \quad deg(z) = 4 \end{cases}$

c) No.

4) (1%) (a) How many vertices and how many edges are there in the complete bipartite graphs $K_{4,7}$, $K_{7,10}$, and $K_{m,n}$, where m, n are positive integers. (b) if $K_{m,6}$ has 72 edges, what is m?

Answer:

a)

	$\lvert V \rvert$	$\lvert E \rvert$
$K_{4,7}$	11	28
$K_{7,10}$	17	70
$K_{m,n}$	$m+n$	$m \cdot n$

b) $m \cdot 6 = 72 \Rightarrow m = 12.$

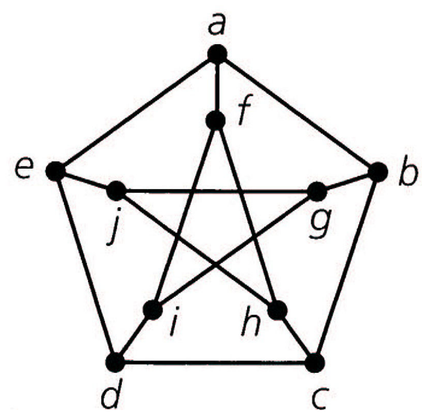
5) (1%) A pet-shop owner receives a shipment of tropical fish. Among the different species are certain pairs where one species eat the other. These pairs must be kept in different tanks. Model this problem as a graph-coloring problem, and tell how to determine the smallest number of tanks need to preserve the fish.

Answer:

Draw a vertex for each species of fish.

If two species x, y must be kept in separate aquaria, draw the edge $\{x, y\}$.

The smallest number of aquaria needed is the chromatic number of the resulting graph.



6) (1%) Show the Petersen graph has a Hamilton path.

   Answer:

   We demonstrate one case to show that the graph does not have Hamilton cycle, but it has a Hamilton path.

   Start at vertex a and consider the partial path $a \to f \to i \to d$. These choices require the removal of edge $\{f, h\}$ and $\{g, i\}$ from further consideration since each vertex of the graph will be incident with exactly two edges in the Hamilton cycle. At vertex d we can go to either vertex c or vertex e.

   (i) If we go to vertex c we eliminate edge $\{e, d\}$ from consideration, but we must now include edges $\{e, j\}$ and $\{e, a\}$, and this forces the elimination of edge $\{a, b\}$. Now we must consider vertex b, for by eliminating edge $\{a, b\}$ we are now required to include edges $\{b, g\}$ and $\{b, c\}$ in the cycle. This forces us to remove edge $\{c, h\}$ from further consideration. But we have now remove edges $\{f, h\}$ and $\{c, h\}$ and there is only one other edge that is incident with h, so no Hamilton cycle can be obtained.

   (ii) Selecting vertex e after d, we remove edge $\{d, c\}$ and include $\{c, h\}$ and $\{b, c\}$. Having removed $\{g, i\}$ we must include $\{g, b\}$ and $\{g, j\}$. This forces the elimination of $\{a, b\}$, the inclusion of $\{a, e\}$ (and the elimination of $\{e, j\}$). We now have a cycle containing a, f, i, d, e, hence this method has also failed.

   However, this graph does have a Hamilton path: $a \to b \to c \to d \to e \to j \to h \to f \to i \to g$.