Name: Student ID:

Quiz #5 (4%)

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

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3:30 - 3:50 p.m., April 15th, 2013

This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (1%) If $A = \{1, 2, 3, 4, 5\}$ and there are 2520 injective functions $f : A \to B$, what is $|B|$?

   Answer:

   $P(|B|, |A|) = P(n, m) = P(n, 5) = \frac{n!}{(n-5)!} = 2520.$

   $|B| = n = 7.$

2) (1%) Answer the following questions.

   a) How many ways can 31,100,905 be factored into three factors, each greater than 1, if the order of the factors is irrelevant?

   b) Answer part (a), assuming the order of the three factors is relevant.

   Answer:

   a) $31,100,905 = 5 \cdot 11 \cdot 17 \cdot 29 \cdot 31 \cdot 37$

      We find that there are $S(6, 3) = 90$ ways.

   b) If the order of the factors is considered relevant.

      There are $(3!)S(6, 3) = 540$ ways.

3) (1%) Let $|A| = 7$, answer the following questions

a) What is $|A \times A|$?

b) How many functions $f : A \times A \to A$ are there?

c) How many closed binary operations are there on A ?

d) How many of these closed binary operations are commutative?

Answer:

a) 49

b) 7^{49}

c) 7^{49}

d) 7^{28}

4) (1%) Let $f, g, h : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = x + 1$, $g(x) = 2x$, $h(x) = 0$ if x is odd, and $h(x) = 1$ if x is even. Determine

   a) $f \circ g$

   b) $h \circ g$

   c) $f \circ (g \circ h)$

   d) g^3

   e) h^{300}

   Answer:

   a) $(f \circ g)(x) = f(g(x)) = f(2x) = 2x + 1$

   b) $(h \circ g)(x) = h(g(x)) = h(2x) = 1$

   c) If x is odd, then $(f \circ (g \circ h))(x) = f(g(h(x))) = f(g(0)) = f(0) = 1$

      If x is even, then $(f \circ (g \circ h))(x) = f(g(h(x))) = f(g(1)) = f(2) = 3$

   d) $g^3(x) = g^2(2x) = g(4x) = 8x$

   e) If x is odd, then $h^{300}(x) = h^{299}(0) = h^{298}(1) = \ldots = h(0) = 1$

      If x is even, then $h^{300}(x) = h^{299}(1) = h^{298}(0) = \ldots = h(1) = 0$