Name: Student ID:

Quiz #6 (4%)

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

Department of Computing Science, National Tsing Hua University, Taiwan

3:30 - 3:50 p.m., May 6th, 2013

This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (1%) For $A = \{a, b, c, d, e, f\}$, each graph, or digraph, in Figure 1 represents a relation $\mathscr{R}$ on A. Write each relation $\mathscr{R} \subseteq A \times A$ as a relation matrix $M(\mathscr{R})$.

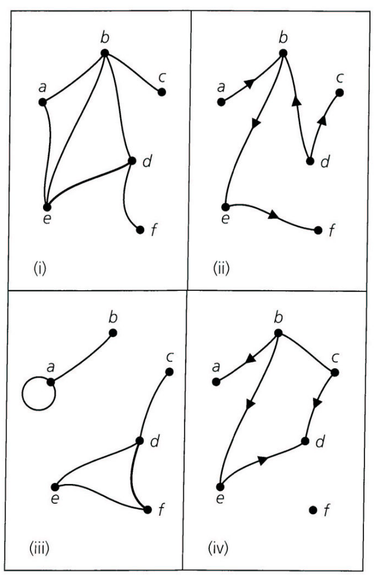
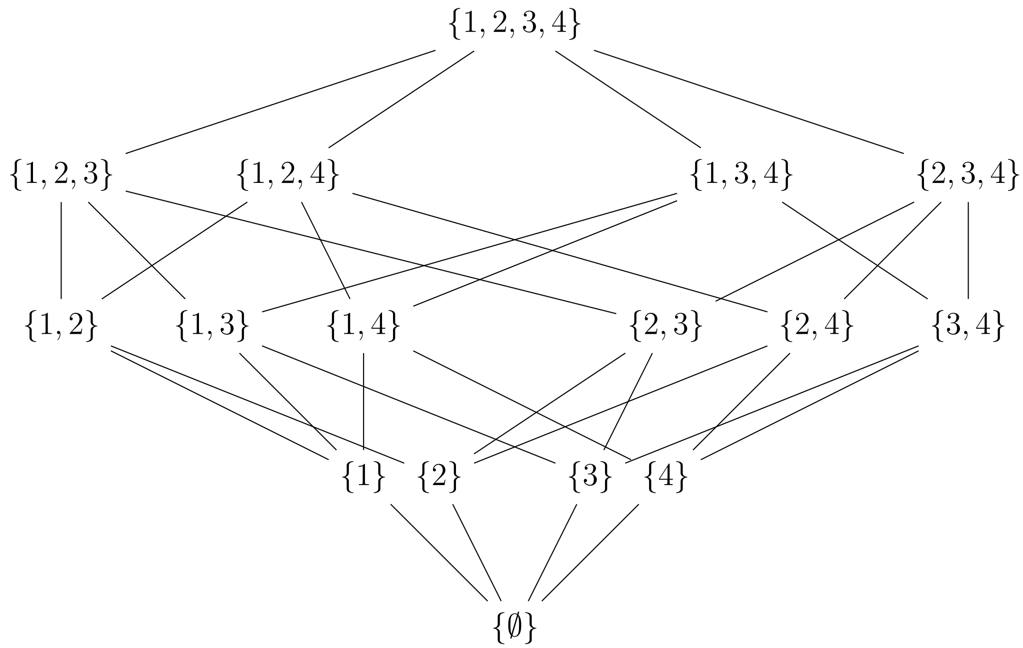


Figure 7.13

$$(i) \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} (ii) \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} (iii) \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} (iv) \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2) (1%) Draw the Hasse diagram for the poset $(\mathscr{P}(\mathscr{U}), \subseteq)$, where $\mathscr{U} = \{1, 2, 3, 4\}$.

Answer:



3) (1%) For $A = \mathbf{R}^2$, define $\mathscr{R}$ on A by $(x_1, y_1)\mathscr{R}(x_2, y_2)$ if $x_1 = x_2$.

a) Verify that $\mathscr{R}$ is an equivalence relation on A.

b) Describe geometrically the equivalence classes and partition of A induced by $\mathscr{R}$.

Answer:

a) i) Reflexive:

$(x, y) \in A$

$\Rightarrow x = x$

$\Rightarrow (x, y)\mathscr{R}(x, y)$

ii) Symmetric:

$(x_1, y_1)\mathscr{R}(x_2, y_2)$

$\Rightarrow x_1 = x_2$

$\Rightarrow (x_2, y_2)\mathscr{R}(x_1, y_1)$

iii) Transitive:

$(x_1, y_1)\mathscr{R}(x_2, y_2)$ and $(x_2, y_2)\mathscr{R}(x_3, y_3)$

$\Rightarrow x_1 = x_2$ and $x_2 = x_3$

$\Rightarrow x_1 = x_3$

$\Rightarrow (x_1, y_1)\mathscr{R}(x_3, y_3)$

$\mathscr{R}$ satisfies reflexive, symmetric and transitive.

$\Rightarrow \mathscr{R}$ is an equivalence relation on A.

b) The whole lines are vertical to x-axis, and they partitions A.

4) (1%) Apply the minimization process to each finite state machine in Figure 4.

	v		ω	
	0	1	0	1
s_1	s_4	s_1	0	1
s_2	s_3	s_3	1	0
s_3	s_1	s_4	1	0
s_4	s_1	s_3	0	1
s_5	s_3	s_3	1	0

(a)

	v		ω	
	0	1	0	1
s_1	s_6	s_3	0	0
s_2	s_5	s_4	0	1
s_3	s_6	s_2	1	1
s_4	s_4	s_3	1	0
s_5	s_2	s_4	0	1
s_6	s_4	s_6	0	0

(b)

Answer:

a) $P_1 : \{s_1, s_4\}, \{s_2, s_3, s_5\}$

$(v(s_1, 0) = s_4)E_1(v(s_4, 0) = s_1)$ but $(v(s_1, 1) = s_1)\neg E_1(v(s_4, 1) = s_3)$

$(v(s_2, 0) = s_3)\neg E_1(v(s_3, 0) = s_1)$

$(v(s_2, 0) = s_3)E_1(v(s_5, 0) = s_3)$ and $(v(s_2, 1) = s_3)E_1(v(s_5, 1) = s_3)$

Since $s_2 \neg E_1 s_3$ and $s_2 E_1 s_5$, it follows that $s_3 \neg E_1 s_5$

Hence P_2 is given by $P_2 : \{s_1\}, \{s_4\}, \{s_2, s_5\}, \{s_3\}$.

$(v(s_2, x) = s_3)E_2(v(s_5, x) = s_3)$ for $x = 0, 1$.

Hence $s_2 E_3 s_5$ and $P_2 = P_3$. Consequently, state s_2 and s_5 are equivalent.

b) States s_2 and s_5 are equivalent.