Name: Student ID:

Quiz #7 (4%)

CS2336 Discrete Mathematics, Instructor: Cheng-Hsin Hsu

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3:30 - 3:50 p.m., May 13th, 2013

This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (1%) Determine the number of integer solutions of $a + b + c + d = 19$, where $-5 \leq a, b, c, d \leq 10$.

   Answer:

   The number of integer solutions for $x_1 + x_2 + x_3 + x_4 = 19$, $-5 \leq x_1 \leq 10$, $1 \leq i \leq 4$, equals the number of integer solutions for $y_1 + y_2 + y_3 + y_4 = 39$, $0 \leq y_i \leq 15$.

   For $1 \leq i \leq 4$, let c_i: $y_i \geq 16$.

   $N(c_i), 1 \leq i \leq 4 : y_1 + y_2 + y_3 + y_4 = 23 : \binom{4+23-1}{23} = \binom{26}{23}$.

   $N(c_i c_j), 1 \leq i < j \leq 4 : y_1 + y_2 + y_3 + y_4 = 7 : \binom{4+7-1}{7} = \binom{10}{7}$.

   $N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4}) = \binom{42}{39} - 4\binom{26}{23} + 6\binom{10}{7}$.

2) (1%)

a) In how many ways can the letters in ARRANGEMENT be arranged so that there are exactly two pairs of consecutive identical letters?

b) Answer part (a), but with at least two pairs of consecutive identical letters?

Answer:

Let c_1 denote the condition that the two A's are together in an arrangement of ARRANGEMENT. Conditions c_2, c_3, c_4 are defined similarly for the two E's, N's, and R's, respectively.

$N = \frac{11!}{(2!)^4} = 2494800.$

For $1 \leq i \leq 4, N(c_i) = \frac{10!}{(2!)^3} = 453600.$

For $1 \leq i < j \leq 4, N(c_i c_j) = \frac{9!}{(2!)^2} = 90720.$

For $1 \leq i < j < k \leq 4, N(c_i c_j c_k) = \frac{8!}{2!} = 20160.$

$N(c_1 c_2 c_3 c_4) = 7! = 5040.$

$S_1 = \binom{4}{1}\frac{10!}{(2!)^3}$, $S_2 = \binom{4}{2}\frac{9!}{(2!)^2}$, $S_3 = \binom{4}{3}\frac{8!}{2!}$, $S_4 = \binom{4}{4}7!$.

a) $E_2 = S_2 - \binom{3}{1}S_3 + \binom{4}{2}S_4 = \binom{4}{2}\frac{9!}{(2!)^2} - \binom{3}{1}\binom{4}{3}\frac{8!}{2!} + \binom{4}{2}7! = 332640.$

b) $L_2 = S_2 - \binom{2}{1}S_3 + \binom{3}{1}S_4 = \binom{4}{2}\frac{9!}{(2!)^2} - \binom{2}{1}\binom{4}{3}\frac{8!}{2!} + \binom{3}{1}7! = 398160.$

3) (1%) How many derangements of 1, 2, 3, 4, 5, 6, 7, 8 start with

a) 1, 2, 3, and 4, in some order?

b) 5, 6, 7, and 8, in some order?

Answer:

a) $N = d_4 \times d_4 = 9^2 = 81$

b) $N = (4!) \times (4!) = (4!)^2 = 576$

4) (1%) Find the rook polynomials for the shaded chessboards in Fig. 1.

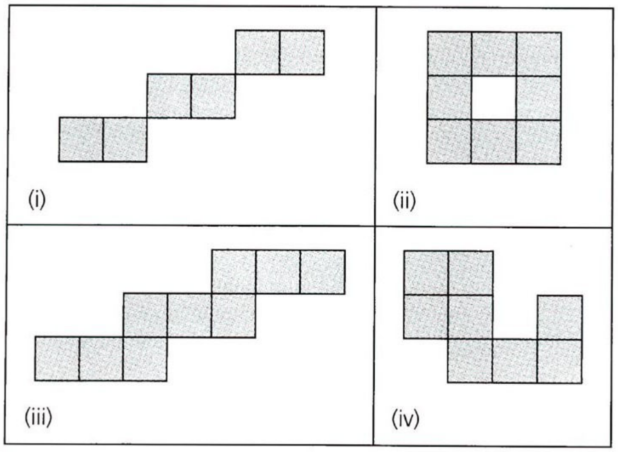

Fig. 1. The chessboards.

(i)$1 + 6x + 12x^2 + 8x^3$

(ii)$1 + 8x + 14x^2 + 4x^3$

(iii)$1 + 9x + 25x^2 + 21x^3$

(iv)$1 + 8x + 16x^2 + 7x^3$