Name: Student ID:

Quiz #8 (4% + 1% Bonus)

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3:30 - 3:50 p.m., May 20th, 2013

This is a closed book test. Any academic dishonesty will automatically lead to zero point.

1) (1%) Find the generating function for the number of ways to select 10 candy bars from large supplies of 6 different kinds.

   Answer:

   $(1 + x^1 + x^2 + x^3 + ... + x^{10})^6$

2) (1%) Find the coefficient of x^{15} in each of the following: (a) $x^3(1-2x)^{10}$ and (b) $(1+x)^4/(1-x)^4$.

   Answer:

   a) 0
   b) $\binom{4}{0}\binom{-4}{15}(-1)^{15} + \binom{4}{1}\binom{-4}{14}(-1)^{14} + \binom{4}{2}\binom{-4}{13}(-1)^{13} + \binom{4}{3}\binom{-4}{12}(-1)^{12} + \binom{4}{4}\binom{-4}{11}(-1)^{11}$
   $= \binom{4}{0}\binom{18}{15} + \binom{4}{1}\binom{17}{14} + \binom{4}{2}\binom{16}{13} + \binom{4}{3}\binom{15}{12} + \binom{4}{4}\binom{14}{11}$

3) (1%) Find the generating function for the number of integer solutions of: (a) $2w + 3x + 5y + 7z = n$, $0 \leq w, x, y, z$ and (b) $2w + 3x + 5y + 7z = n$, $0 \leq w$, $4 \leq x, y$, $5 \leq z$.

   Answer:

   a) $\frac{1}{1-x^2} \cdot \frac{1}{1-x^3} \cdot \frac{1}{1-x^5} \cdot \frac{1}{1-x^7}$
   b) $\frac{1}{1-x^2} \cdot \frac{x^{12}}{1-x^3} \cdot \frac{x^{20}}{1-x^5} \cdot \frac{x^{35}}{1-x^7}$

4) (1%) If a 20-digit ternary (0, 1, 2) sequence is randomly generated, what is the probability that: (a) it has an even number of 1's? and (b) The total number of 0's and 1's is even?

   Answer:

   a) $\frac{1}{2} \cdot \frac{3^{20}+1}{3^{20}}$
   b) $\frac{1}{2} \cdot \frac{3^{20}+1}{3^{20}}$

5) (1%) Let $f(x)$ be the generating function for the sequence $a_0, a_1, a_2, \ldots$. What is the sequence corresponding to the generating function $(1-x)f(x)$?

   Answer:

$$f(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n + ...$$

$$(1-x)f(x) = (1-x)(a_0 + a_1 x + a_2 x^2 + ... + a_n x^n + ...)$$

$$= a_0 + (a_1 - a_0)x + (a_2 - a_1)x^2 + ... + (a_n - a_{n-1})x^n + ...$$

   The sequence is $a_0, a_1 - a_0, a_2 - a_1, ..., a_n - a_{n-1}, ...$