Solution

Ex 3.1: 2, 5, 10, 15, 29

Ex 3.2: 2, 4, 7, 17, 19

Ex 3.3: 4, 5, 6, 10

Ex 3.4: 4, 8, 9, 11, 15

Ex 3.1: (2)

* All of the statement are true except for part (f). Because the element 2 doesn’t exist in A.

Ex 3.1: (5)

a) {0, 2}
b) {2, 2(1/2), 3(1/3), 5(1/5), 7(1/7)}
c) {0, 2, 12, 36, 80}

Ex 3.1: (10)

- The nonempty sets are in parts (d), and (f).

Ex 3.1: (15)

- $W = \{1\} \in \{\{1\}, 2\} = X$
- $X = \{\{1\}, 2\} \in \{X, 3\} = Y$
- $W = \{1\} \notin \{X, 3\} = Y$

Ex 3.1: (29)

* **procedure** Subsets (i, j, k, l: positive integars)

```
begin
    for i := 1 to 4 do
        for j := i+1 to 5 do
            for k = j+1 to 6 do
                for l := k+1 to 7 do
                    print ({i, j, k, l})
end
```

Ex 3.2: (2)

a) [2,3]
b) [0,7)
c) (-∞,0) ∪ (3, +∞)
d) [0,2) ∪ (3,7)
e) [0,2)
f) (3,7)

Ex 3.2: (4)

a) True: (i), (iv), (v)
b) (i) E (ii) B (iii) D (iv) D
(v) Z-A={2n+1|n∈Z} (vi) E

Ex 3.2: (7)

a) False. $Let\ \mathcal{U} = \{1,2,3\}, A = \{1\}, B = \{2\}, C = \{3\}.$
$Then\ A \cap B = B \cap C\ but\ A \neq B.$

b) False. $Let\ \mathcal{U} = \{1,2\}, A = \{1\}, B = \{2\}, C = \{1,2\}.$
$Then\ A \cup B = A \cup C\ but\ A \neq B.$

c) $x \in A \Longrightarrow x \in A \cup C \Longrightarrow x \in B \cup C. So\ x \in B\ or\ x \in C. If\ x \in B,$
$then\ we\ are\ finished. If\ x \in C, then\ A \cap C = B \cap C\ and\ x \in B.$
$In\ either\ case, x \in B\ so\ A \subseteq B.$
$Likewise, y \in B \Longrightarrow y \in B \cup C = A \cup C, so\ y \in A\ or\ y \in C.$
$If\ y \in C, then\ y \in B \cap C = A \cap C.$
$In\ either\ case, y \in A\ and\ B\ \subseteq A. Hence\ A = B.$

d) Let $x \in A$. Consider two cases:

1. $x \in C \Longrightarrow x \notin A \triangle C \Longrightarrow x \notin B \triangle C \Longrightarrow x \in B.$
2. $x \notin C \Longrightarrow x \in A \triangle C \Longrightarrow x \notin B \triangle C \Longrightarrow x \in B.$

In either case $A \subseteq B$. In a similar way we find $B \subseteq A$, so $A = B$

Ex 3.2: (17)

a) $A \cap (B - A) = A \cap \left(B \cap \overline{A}\right) = B \cap \left(A \cap \overline{A}\right) = B \cap \emptyset = \emptyset.$

b) $\left[(A \cap B) \cup \left(A \cap B \cap \overline{C} \cap D\right)\right] \cup \left(\overline{A} \cap B\right)$

$= (A \cap B) \cup \left(\overline{A} \cap B\right)$, *by the Absorption*

$= \left(A \cup \overline{A}\right) \cap B = \mathcal{U} \cap B = B$

c) $(A - B) \cup (A \cap B) = \left(A \cap \overline{B}\right) \cup (A \cap B) = A \cap \left(\overline{B} \cup B\right) = A \cap \mathcal{U} = A$

d) $\overline{A} \cup \overline{B} \cup \left(A \cap B \cap \overline{C}\right) = \overline{(A \cap B)} \cup \left[(A \cap B) \cap \overline{C}\right] =$
$\left[\overline{(A \cap B)} \cup (A \cap B)\right] \cap \left[\overline{(A \cap B)} \cup \overline{C}\right] = \left[\overline{(A \cap B)} \cup \overline{C}\right] = \overline{A} \cup \overline{B} \cup \overline{C}$

Ex 3.2: (19)

a) [-6, 9]
b) [-8, 12]
c) Ø
d) [-8, -6) ∪ (9, 12]
e) [-14, 21]
f) [-2, 3]
g) **R**
h) [-2, 3]

Ex 3.3: (4)

a) Here $A \cup B \cup C = C$, so $|A \cup B \cup C| = |C| = 5000$.

b) Here $A \cap B \cap C = \emptyset$ as well, so it follows from the formula for $|A \cup B \cup C| = |A| + |B| + |C| = 50 + 500 + 5000 = 5550$.

c) $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| = 5000 - 3 - 3 - 3 + 1 = 5542$

Ex 3.3: (5)

- $9! + 9! - 8!$

Ex 3.3: (6)

a) 12
b) 2
c) 16

Ex 3.3: (10)

- The number of arrangements with either H before E, or E before T, or T before M equals the total number of arrangements (i.e., 7!) minus the number of arrangements where E is before H, and T is before E and M is before T. There are 3! ways to arrange C, I, S. For each arrangement there are four locations (one at the start, two between pairs of letters, and one at the end) to select from, with repetition, to place M, T, E, H in this prescribed order. Hence there are $(3!)\binom{4+4-1}{4} = (3!)\binom{7}{4}$ arrangements where M is before T, T before E, and E before H. Consequently, there are $7! - 3!\binom{7}{4}$ arrangements with either H before E, or E before T, or T before M.

Ex 3.4: (4)

- The probability of each equally likely outcome is $\frac{0.14}{7} = 0.02 = \frac{1}{n}$.
- Therefore, $n = \frac{1}{0.02} = 50$.

Ex 3.4: (8)

* $S = \{\{a, b, c\} | a, b, c \in \{1,2,3, \dots, 99,100\}, a \neq b, a \neq c, b \neq c\}$
* $A = \{\{a, b, c\} | \{a, b, c\} \in S, a + b + c \ is \ even\}$
  $= \{\{a, b, c\} | \{a, b, c\} \in S, a, b, c \ are \ even,$
  $or \ one \ of \ a, b, c \ is \ even \ and \ the \ other \ two \ integers \ are \ odd\}$
* $|S| = \binom{100}{3} = 161{,}700; |A| = \binom{50}{3} + \binom{50}{1}\binom{50}{2} = 80{,}850$

$$\Pr(A) = \frac{1}{2}$$

Ex 3.4: (9)

* The sample space $S = \{(x_1, x_2, \ldots, x_6) | xi = H \; or \; T, 1 \leq i \leq 6\}$. Hence $|S| = 2^6 = 64$.

a) The event $A = \{HHHHHH\}$ and $\Pr(A) = 1/64$.

b) The event $B = \{TTTTTH, TTTTHT, TTTHTT, TTHTTT, THTTTT, HTTTTT\}$ and $\Pr(B) = 3/32$.

c) There are $\frac{6!}{4!2!} = 15$ ways. The probability is $15/64$.

d) $(0 \times H)$:1 ; $(2 \times H)$:$\frac{6!}{4!2!} = 15$; $(4 \times H)$:$\frac{6!}{2!4!} = 15$; $(6 \times H)$:1
The probability is $(1 + 15 + 15 + 1)/64 = 1/2$

e) $(4 \times H)$:$\frac{6!}{2!4!} = 15$; $(5 \times H)$:$\frac{6!}{5!1!} = 6$; $(6 \times H)$:1
The probability is $(15 + 6 + 1)/64 = 11/32$

Ex 3.4: (11.a)

Let $S = the\ sample\ space$
$= \{(x_1, x_2, x_3)|1 \leq x_i \leq 6, i = 1,2,3\}$; $|S| = 63 = 216$.
Let $A = \{(x_1, x_2, x_3)|x_1 < x_2\ and\ x_1 < x_3\}$
$= \cup_{n=1}^{5}\{(n, x_2, x_3)|n < x_2\ and\ n < x_3\}$.
For $1 < n < 5, |\{(n, x_2, x_3)|n < x_2\ and\ n < x_3\}| = (6 - n)2$.
Consequently, $|A| = 52 + 42 + 3^2 + 2^2 + 1^2 = 55$.
Therefore, $\Pr(A) = 55/213$

Ex 3.4: (11.b)

With S as in part (a), let $B = \{(x_1, x_2, x_3)|x_1 < x_2 < x_3\}$.

Then $|\{(1, x_2, x_3)|1 < x_2 < x_3\}| = 10,$

$|\{(2, x_2, x_3)|2 < x_2 < x_3\}| = 6,$

$|\{(3, x_2, x_3)|3 < x_2 < x_3\}| = 3,$

$|\{(4, x_2, x_3)|4 < x_2 < x_3\}| = 1,$

$so\ |B| = 20\ and\ \Pr(B) = 20/216 = 5/54$

Ex 3.4: (15)

* $\Pr(A) = \frac{1}{3}, \Pr(B) = \frac{7}{15}, \Pr(A \cap B) = \frac{2}{15}, \Pr(A \cup B) = \frac{2}{3}.$
* $\Pr(A \cup B) = \frac{2}{3} = \frac{1}{3} + \frac{7}{15} - \frac{2}{15} = \Pr(A) + \Pr(B) - \Pr(A \cap B).$