

# SageMath 1: Using SageMath



**Cheng-Hsin Hsu**

*National Tsing Hua University*

*Department of Computer Science*

# What is SageMath

- Sage stands for *System for Algebra and Geometry Experimentation*
- Starts from a **Computer Algebra System (CAS)**
- Now a full spectrum of math tools, including
  - **Algebra**
  - Combinatorics
  - Matrix Computations
  - Numerical Math
  - Calculus
  - **Number Theory**
  - Visualization
  - Many other topics



<http://www.sagemath.org>

# How is SageMath Built

---

- Created in 2005 by William Stein at University of Washington
- Goal: Offer an affordable math software to solve various math problems
- Approach
  - Leverage **opensource** (GPL-ed) software, such as *SciPy, Maxima, and GAP*
  - Adopt the popular **Python** as the programming language
- **Outcome: A huge set of math tools!**

# What SageMath Can Do

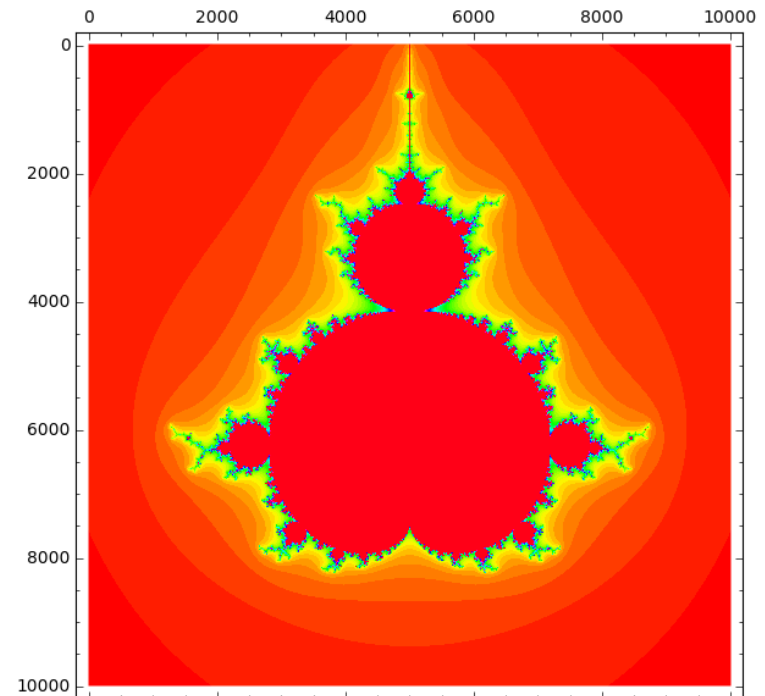
- Solving exact problems

```
⊕ ⓘ  
integral(x*sin(x^2),x)  
-1/2*cos(x^2)  
⊕ ⓘ  
integral(x*sin(x^2),x, -1, 1)  
0
```

- Solving numerical problems

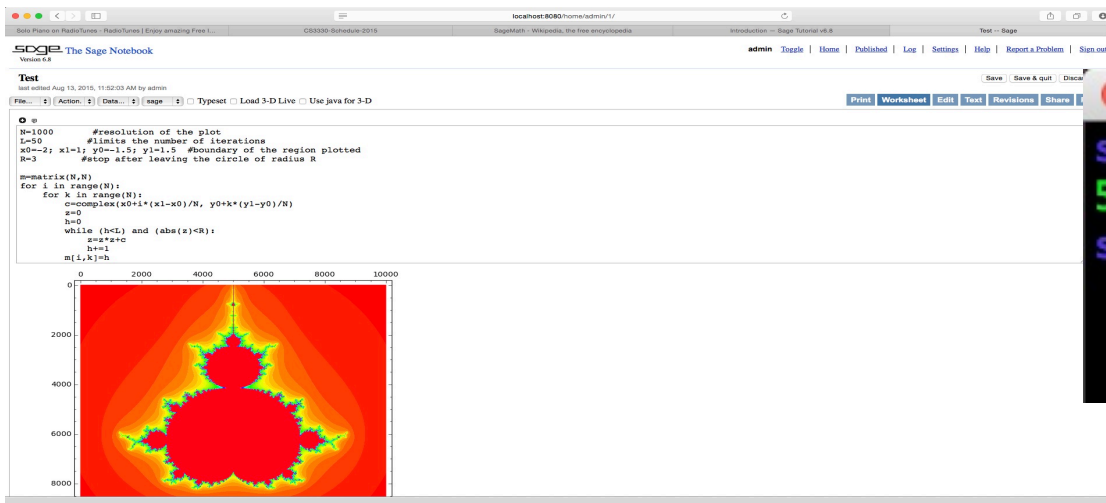
```
⊕ ⓘ  
numerical_integral(x*sin(x^2), -1, 1)  
(0.0, 5.103670946680643e-15)
```

- Plot cool figures
  - Get a sense of more global behavior



# Different Ways to Use SageMath

- Notebook: Web-based interface
- SageMath Online: Cloud-based Web interface
- Interactive command-line: Python-like shell prompt
- Programs: Write and compile Sage programs
- Scripts: Invoke Sage libraries from Python



The screenshot shows a terminal window with a dark background. The prompt 'sage:' is shown in blue. The user has entered the command 'number\_of\_partitions(4)' in green, and the output '5' is shown in green. The prompt 'sage:' is shown again in blue, followed by a white cursor.

```
sage: number_of_partitions(4)
5
sage: 
```

# Where Are Notebook and Command-line

start here

The screenshot shows a Beamer presentation titled "SageMath 1: Using SageMath" displayed in a Mac OS window. The presentation content includes the presenter's name, affiliation, and a list of topics. A red box highlights the "Terminal Session" option in the "SageMath" menu. Another red box highlights the SageMath icon in the dock.

**SageMath 1: Using SageMath**

Cheng-Hsin Hsu  
National Tsing Hua University  
Department of Computer Science

What is SageMath

- Sage stands for *System for Algebra and Geometry Experimentation*
- Starts from a **Computer Algebra System (CAS)**
- Now a full spectrum of math tools, including
  - algebra
  - Combinatorics
  - Matrix Computations
  - Numerical Math
  - Calculus
  - Number Theory
  - Visualization
  - Many other topics

How is SageMath Built

- Created in 2005 by William Stein at University of Washington
- Goal: Create an affordable math software to solve various math problems
- Approach
  - Leverage **opensource (GPL-ed)** software, such as **SOPL, Maxima, and GUP**
  - Adopt the popular **Python** as the programming language
- Outcome: **A huge set of math tools!**

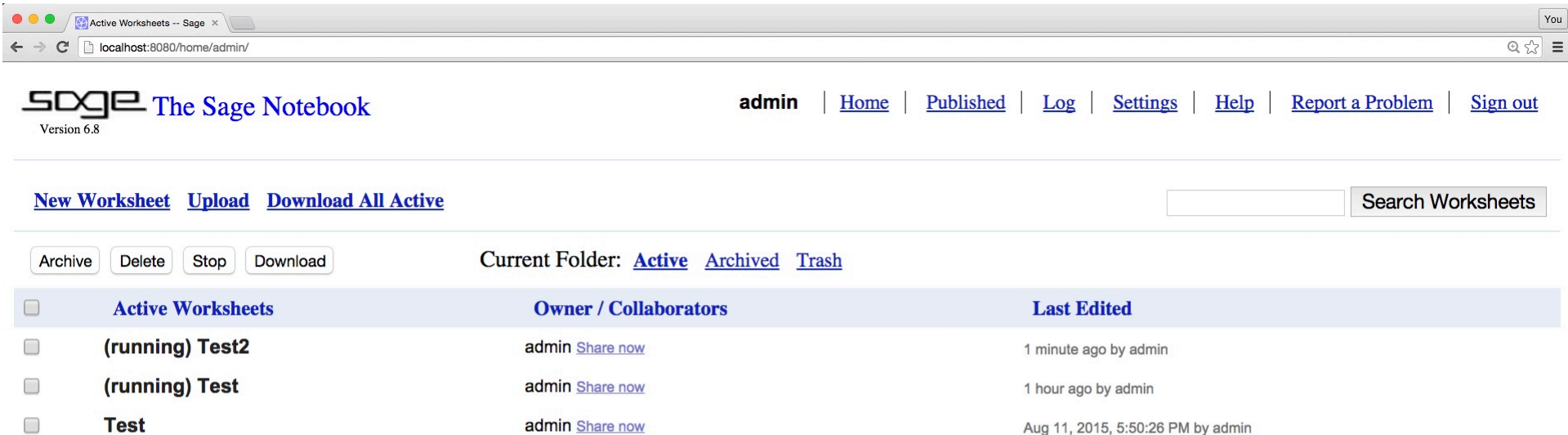
What SageMath Can Do

- Solving exact problems
- Solving numerical problems
- Plot cool figures
  - Get a sense of more global behavior

How Do We Use SageMath

start here

# SageMath Notebook



The screenshot shows the SageMath Notebook interface in a web browser. The browser address bar shows 'localhost:8080/home/admin/'. The page header includes the SageMath logo, 'The Sage Notebook', and 'Version 6.8'. The user is logged in as 'admin'. The navigation menu includes links for 'Home', 'Published', 'Log', 'Settings', 'Help', 'Report a Problem', and 'Sign out'. Below the navigation menu, there are links for 'New Worksheet', 'Upload', and 'Download All Active'. A search box labeled 'Search Worksheets' is present. The main content area shows a table of active worksheets with columns for 'Active Worksheets', 'Owner / Collaborators', and 'Last Edited'. The table lists three worksheets: '(running) Test2', '(running) Test', and 'Test'.

Active Worksheets	Owner / Collaborators	Last Edited
<input type="checkbox"/> (running) Test2	admin <a href="#">Share now</a>	1 minute ago by admin
<input type="checkbox"/> (running) Test	admin <a href="#">Share now</a>	1 hour ago by admin
<input type="checkbox"/> Test	admin <a href="#">Share now</a>	Aug 11, 2015, 5:50:26 PM by admin

- Each notebook has several **worksheets**
- Worksheets can be saved, opened, downloaded, shared, and etc.
- SageMath Online: <http://cloud.sagemath.com/> provides similar interface

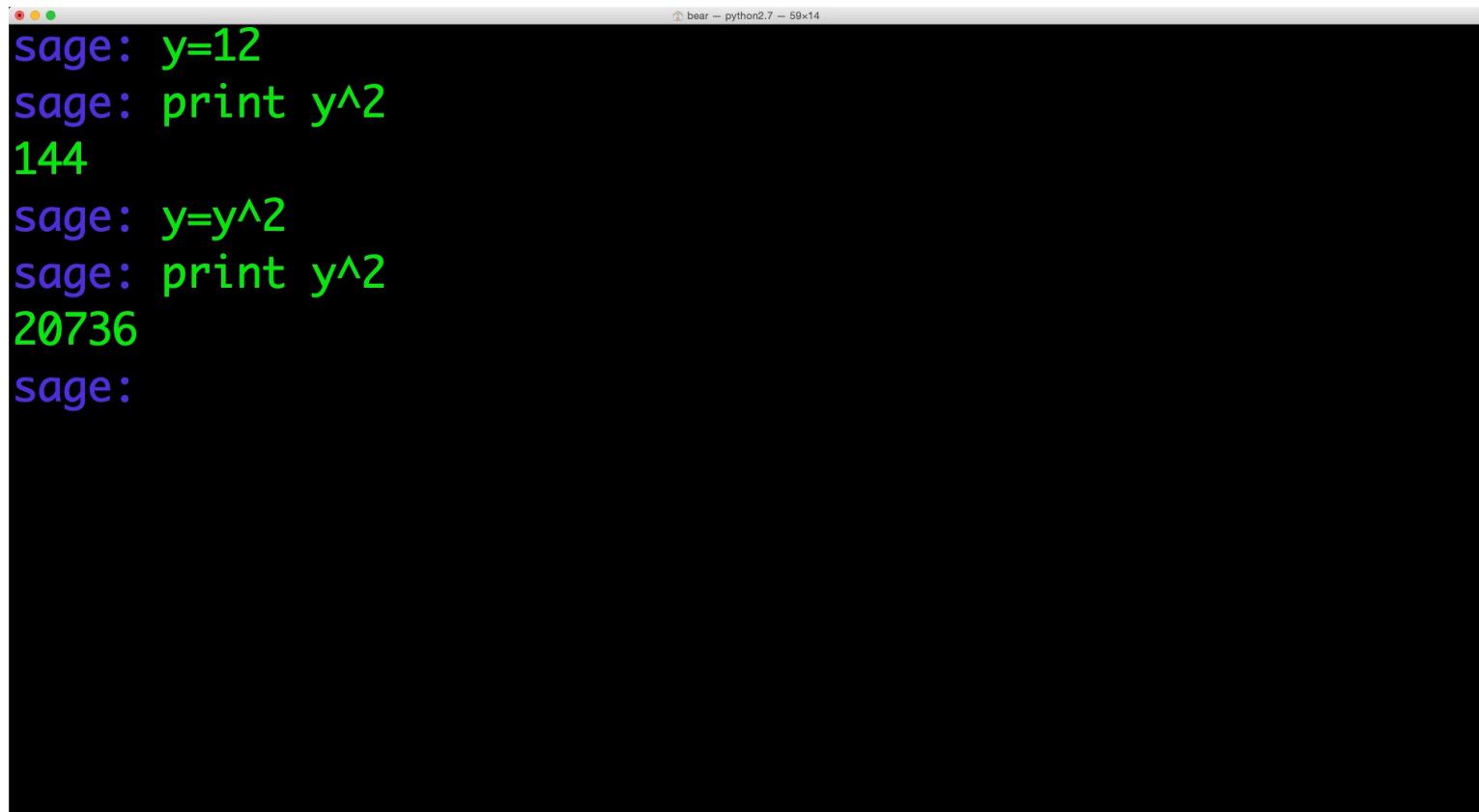
# SageMath Worksheet

The screenshot shows the SageMath Notebook interface. At the top, there's a navigation bar with the Sage logo and "The Sage Notebook" text, along with a version number "Version 6.8". To the right of the logo are several links: "admin", "Toggle", "Home", "Published", "Log", "Settings", "Help", "Report a Problem", and "Sign out". Below the navigation bar, the main content area is titled "Test" and shows the last edited time: "last edited Aug 11, 2015, 5:50:26 PM by admin". There are three buttons: "Save", "Save & quit", and "Discard & quit". Below this, there's a toolbar with buttons for "File...", "Action...", "Data...", "sage", "Typeset", "Load 3-D Live", "Use java for 3-D", "Print", "Worksheet", "Edit", "Text", "Revisions", "Share", and "Publish". The main workspace contains two cells. The first cell has a plus icon and a minus icon, and contains the command `n(sqrt(123))` and its output `11.0905365064094`. The second cell also has plus and minus icons and contains the command `arccos(1/3)`, which is highlighted with a blue border. Below the second cell is an "evaluate" button.

- Each worksheet contains several (independent, stateless) **cells**
- Each cell is a box where you can type in formulas and **evaluate** them ← by pressing the **evaluate** button or **Shift-Enter**
- You can always come back and edit your formulas/commands and **re-evaluate** them



# SageMath Commandline



```
sage: y=12
sage: print y^2
144
sage: y=y^2
sage: print y^2
20736
sage:
```

The screenshot shows a terminal window with a black background and green text. The window title bar at the top reads "bear - python2.7 - 59x14". The terminal content shows a sequence of SageMath commands and their outputs: "sage: y=12" (no output), "sage: print y^2" (output: "144"), "sage: y=y^2" (no output), "sage: print y^2" (output: "20736"), and "sage:" (no output).

- Command-line remembers states

# Use SageMath as a Calculator



## Introduction

Save Save & quit Discard & quit

last edited Aug 13, 2015, 3:28:17 PM by admin

File... Action... Data... sage  Typeset  Load 3-D Live  Use java for 3-D

Print Worksheet Edit Text Revisions Share Publish



2+3

5



900\*(1+0.05\*(90/365))

911.095890410959



10000\*(1+0.12)^3

14049.2800000000



evaluate



# Square Root Function

Introduction Save Save & quit Discard & quit

last edited Aug 13, 2015, 3:28:17 PM by admin

File... Action... Data... sage  Typeset  Load 3-D Live  Use java for 3-D

Print Worksheet Edit Text Revisions Share Publish

`sqrt(25)`  
5

`sqrt(8)`  
 $2\sqrt{2}$  ← SageMath loves exact answers

`N(sqrt(8))`  
2.82842712474619 ← Numerical approximation

`64^(1/6)`  
2

# Compared to Matlab

---

- Everything is double precision floating point by default
- Through Symbolic toolbox, which is a trimmed down version of Maple:

<http://www.maplesoft.com/products/Maple/>

# Special Constants

$$1.2345 = \underbrace{12345}_{\text{mantissa}} \times 10^{\underbrace{-4}_{\text{exponent}}}$$



## Introduction

last edited Aug 13, 2015, 3:28:17 PM by admin

File... Action... Data... sage  Typeset  Load 3-D Live  Use java for 3-D

Print Worksheet Edit Text Revisions Share Publish



```
pi*2
```

```
2*pi
```



**Simplest exact value**



```
e^2
```

```
e^2
```



```
e^2.5
```

```
12.1824939607035
```



**Decimals automatically lead to numerical approximation**



```
N(pi, prec=256)
```

```
3.1415926535897932384626  
06286
```

**prec: (integer) default = 53: prec is the number of bits used to represent the mantissa of a floating-point number.**



# More on Square Roots

The screenshot shows a SageMath web interface with the following elements:

- Page Title:** Introduction
- Metadata:** last edited Aug 13, 2015, 3:28:17 PM by admin
- Navigation:** Save, Save & quit, Discard & quit
- Tools:** File..., Action., Data..., sage,  Typeset,  Load 3-D Live,  Use java for 3-D
- Actions:** Print, Worksheet, Edit, Text, Revisions, Share, Publish
- Code Input 1:** `sqrt(9)`
- Output 1:** `3`
- Code Input 2:** `sqrt(9, all=True)`
- Output 2:** `[3, -3]` (highlighted with a red arrow and text: **List in Python/Sage, a sequence of comma-separated numbers**)
- Code Input 3:** `sqrt(-9, all=True)`
- Output 3:** `[3*I, -3*I]`
- Code Input 4:** (empty)
- Buttons:** evaluate

# SageMath is Case Sensitive

The screenshot shows a web browser window with the URL `localhost:8080/home/admin/3/`. The page title is "Introduction" and it was last edited on Aug 13, 2015, at 3:28:17 PM by admin. The interface includes a toolbar with buttons for "File...", "Action.", "Data...", "sage", "Typeset", "Load 3-D Live", and "Use java for 3-D". There are also buttons for "Print", "Worksheet", "Edit", "Text", "Revisions", "Share", and "Publish".

The main content area displays four code blocks with their outputs:

- Block 1: Input `sQrt(9)`. Output: `Traceback (click to the left of this block for traceback)`, `...`, `NameError: name 'sQrt' is not defined`. A red annotation next to the error message reads: **Mostly, with four exceptions: *true, false, i, n***.
- Block 2: Input `true`. Output: `True`.
- Block 3: Input `3*i - 3*I`. Output: `0`.
- Block 4: Input `n(sqrt(17)) - N(sqrt(13))`. Output: `0.517554350153671`.

# How to Get Help

## Introduction

last edited Aug 13, 2015, 3:28:17 PM by admin

File... Action... Data... sage  Typeset  Load 3-D Live  Use java for 3-D

Print Worksheet Edit

+ ⌘

log?

**File:** /Applications/Sage-6.8.app/Contents/Resources/sage/local/lib/python2.

**Type:** <class 'sage.functions.log.Function\_log'>

**Definition:** log(\*args, \*\*kwds)

**Docstring:**

The natural logarithm of x. See *log?* for more information about its beh

```
Last login: Thu Aug 13 16:35:13 on ttys0
Bears-iMac:~ bear$ /Applications/Sage-6.
| SageMath Version 6.8, Release Date: 20
| Type "notebook()" for the browser-base
| Type "help()" for help.
sage: help(log)
sage: 
```

- Notebook: *command?*
- Command-line: *help(command)*
- Also, utilize the *Tab-completion*



# Trigonometry

Introduction (Sage) x You

localhost:8080/home/admin/3/

**Introduction** Save Save & quit Discard & quit

last edited Aug 13, 2015, 3:28:17 PM by admin

File... Action... Data... sage  Typeset  Load 3-D Live  Use java for 3-D

Print Worksheet Edit Text Revisions Share Publish

$\sin(\pi/3)$

$1/2*\sqrt{3}$

$N(\cos(\pi/12))$   $N(\cos(\pi/6),\text{digits}=16) - 2.0*N(\sin(\pi/3),\text{digits}=16)$

$0.965925826289068$   $-0.866025403784439$

$\arccos(1/2)$   $N(\cos(\pi/6)-2*\sin(\pi/3),\text{digits}=16)$

$1/3*\pi$   $-0.8660254037844386$

$\cos(\pi/6)-\sin(\pi/3)$

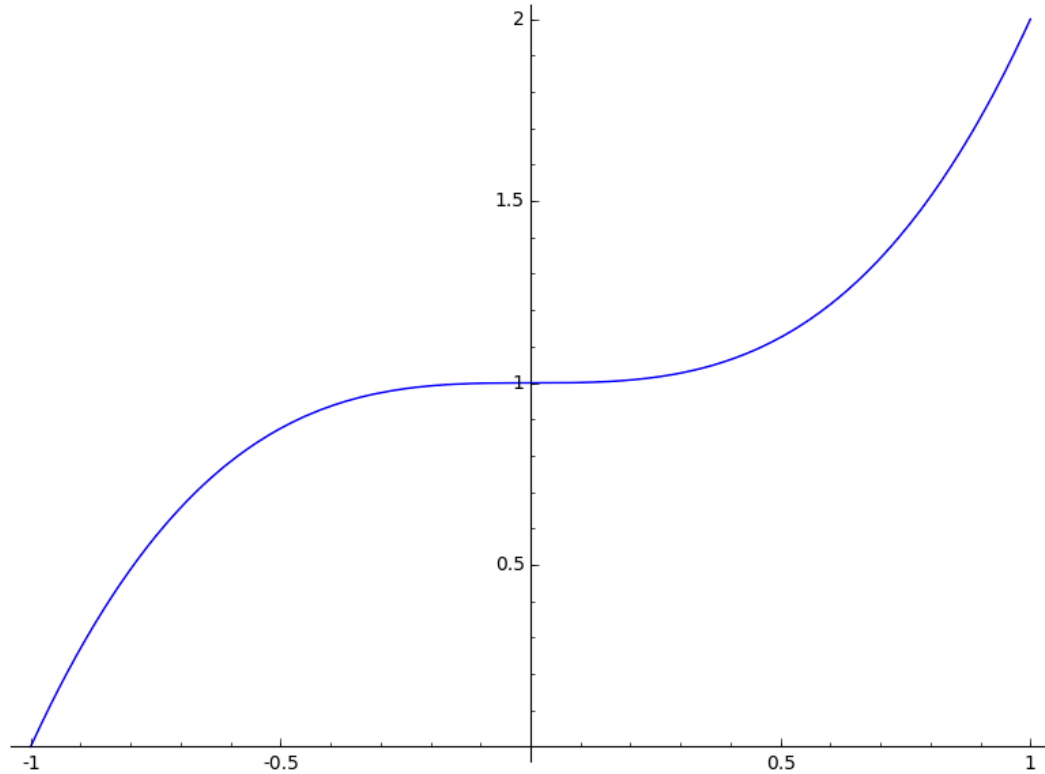
$0$

# Simple 2D Plots



```
plot(x^3+1)
```

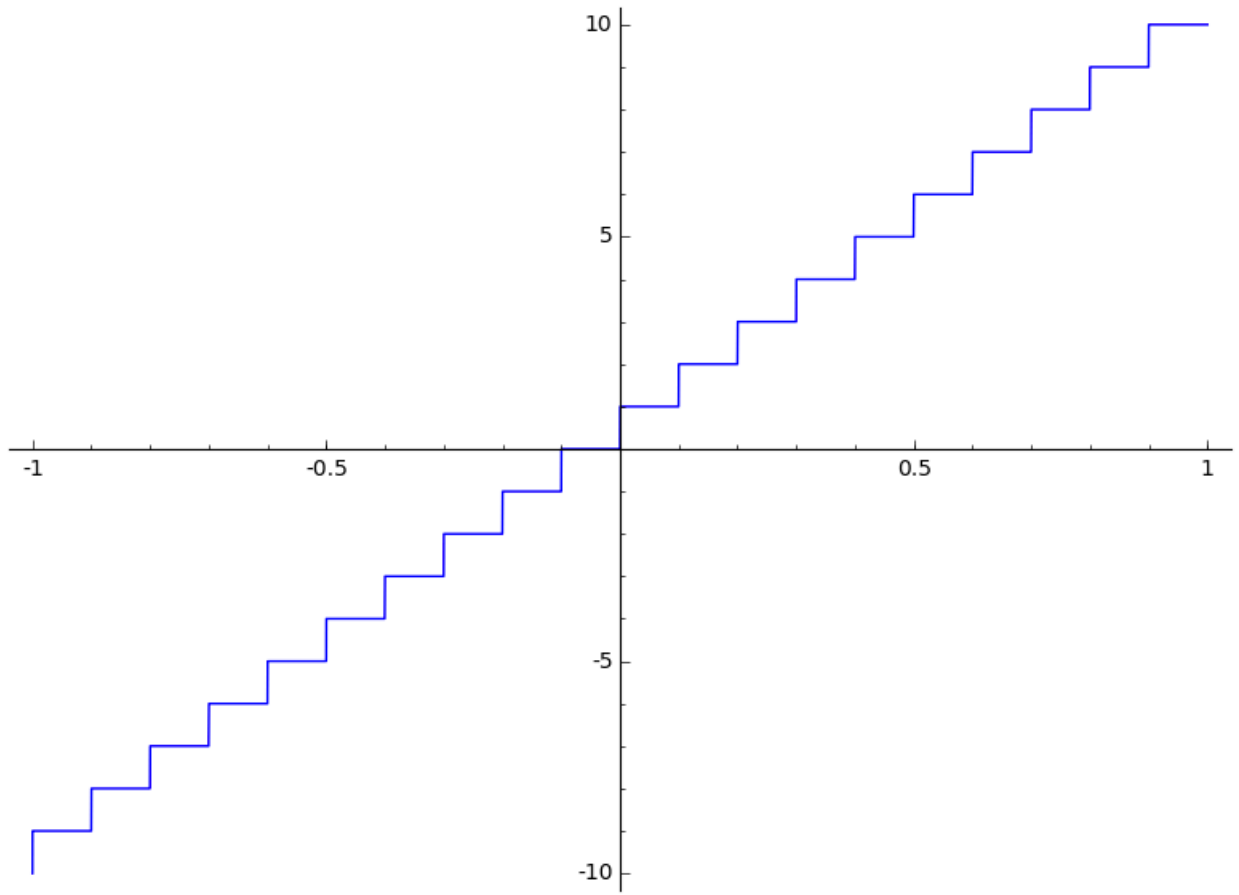
evaluate



# More Complicated Function



```
plot(ceil(x*10))
```

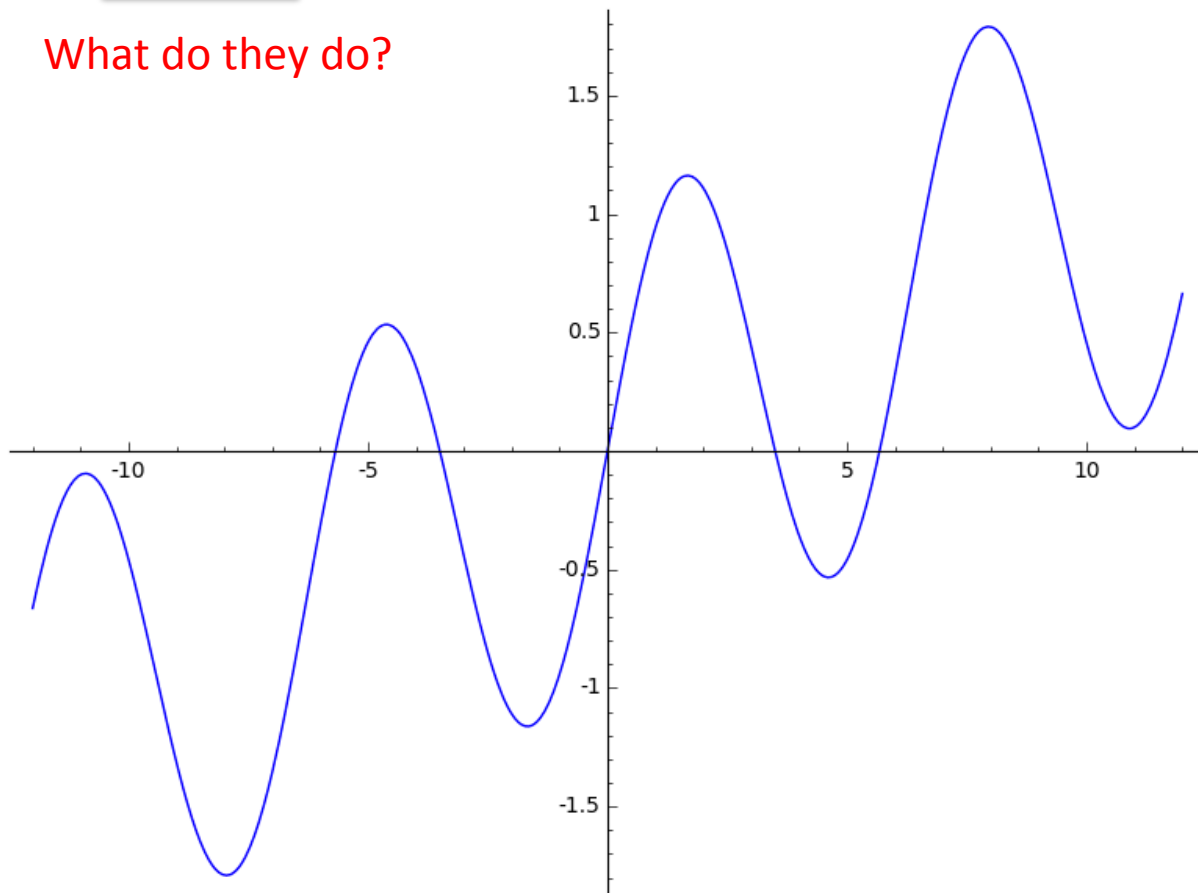


# Even More Complicated



```
plot(sin(x)+0.1*x, -12, 12)
```

What do they do?

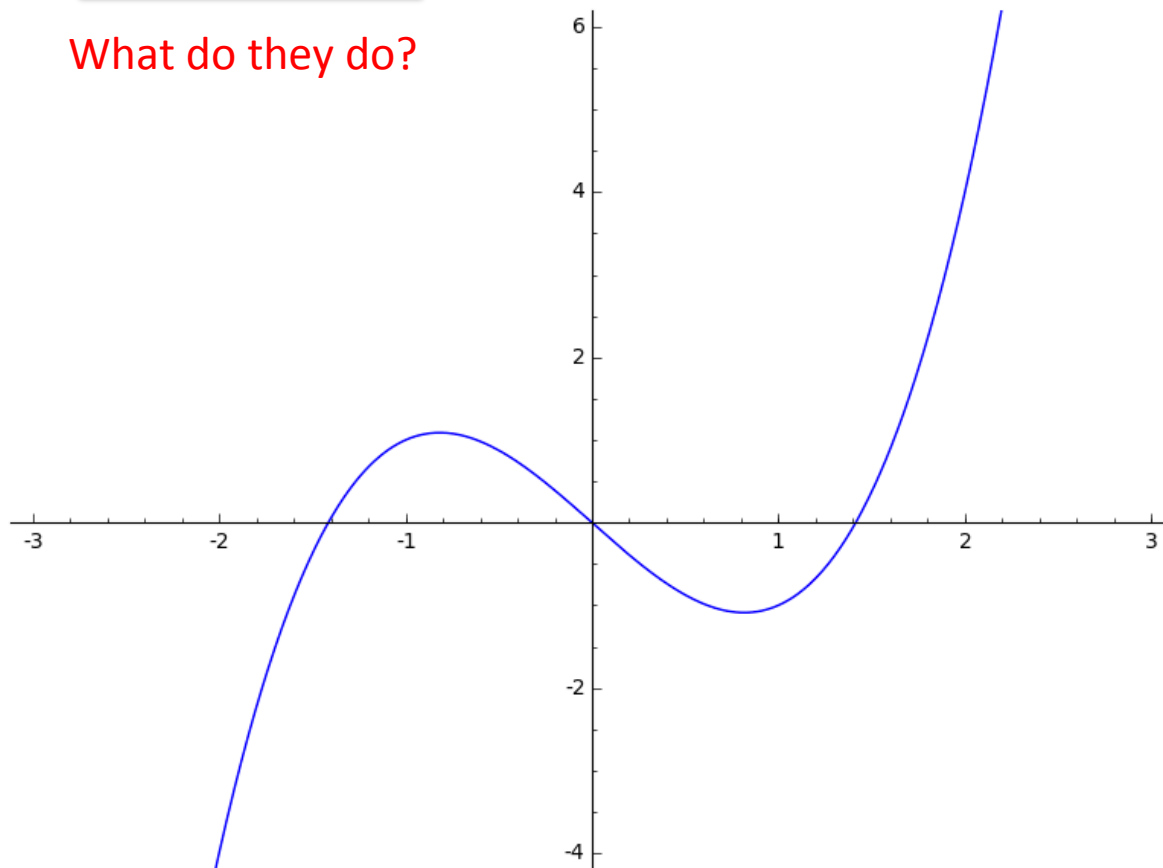


# Ranges of Axes



```
plot(x^3-2*x, -3, 3, ymin=-4, ymax=6)
```

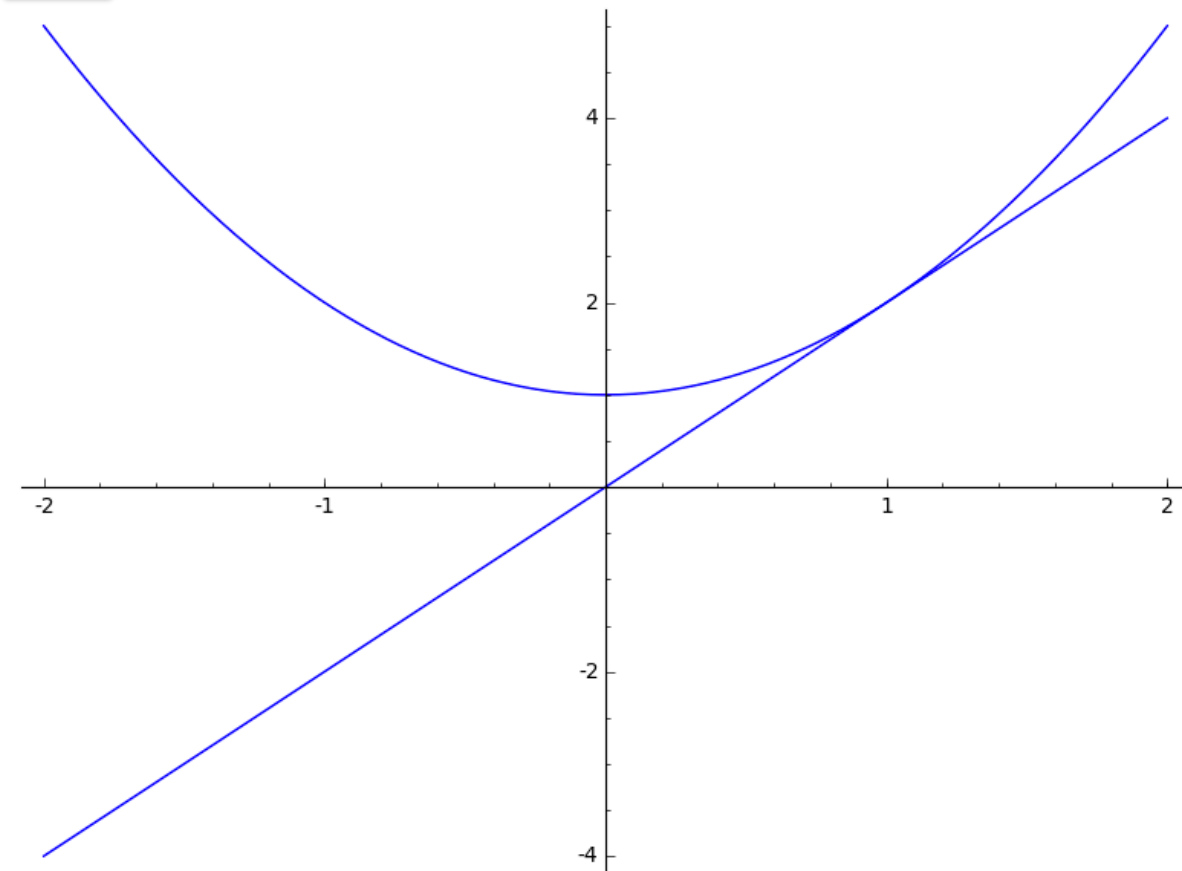
What do they do?



# Superimposing of Multiple Graph



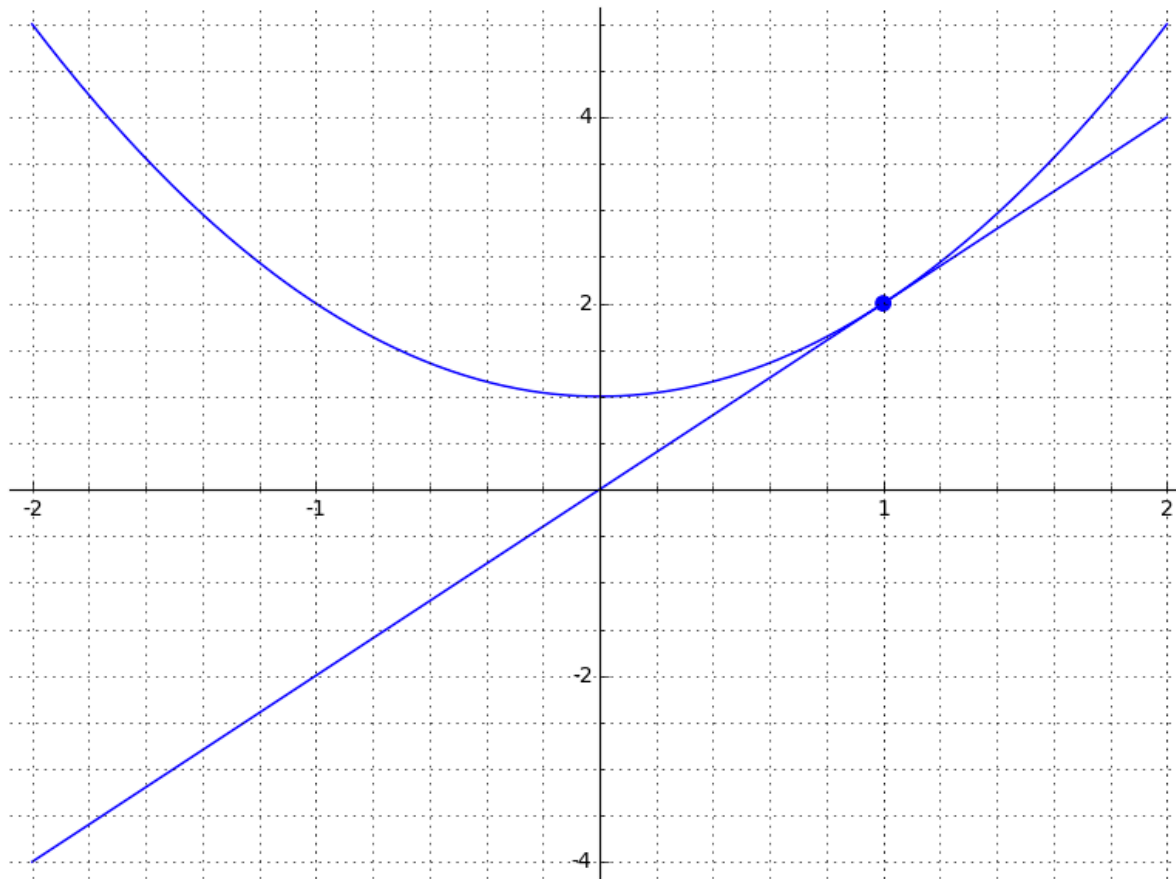
```
plot(2*x, -2, 2) + plot(x^2+1, -2,2)
```



# Polished Graph



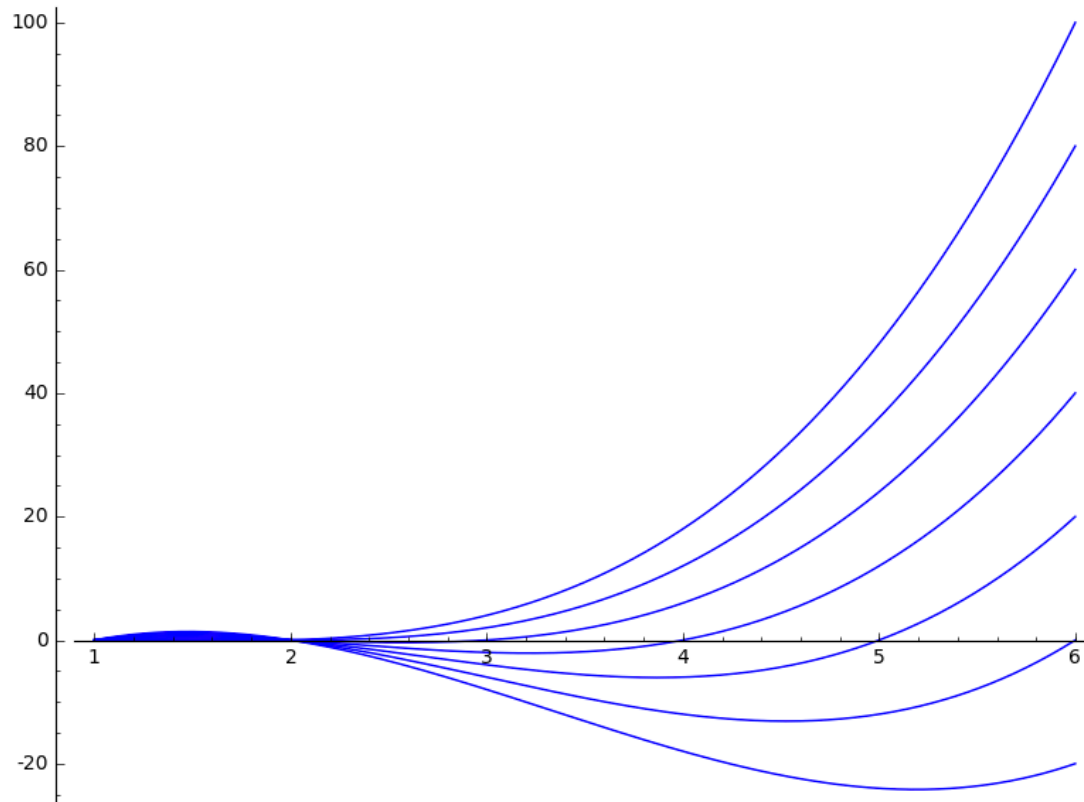
```
plot(2*x, -2, 2, gridlines='minor') + plot (x^2+1, -2,2) + point ((1,2), size=60)
```



# Last Example on Graphs



```
plot((x-1)*(x-2)*(x-7), 1,6)+plot((x-1)*(x-2)*(x-6), 1,6)+plot((x-1)*(x-2)*(x-5),  
1,6)+plot((x-1)*(x-2)*(x-4), 1,6)+plot((x-1)*(x-2)*(x-3), 1,6)+plot((x-1)*(x-2)*(x-2),  
1,6)+plot((x-1)*(x-2)*(x-1), 1,6)
```





# Reduced Row Echelon Form (RREF)

- System of equations:

$$\begin{aligned} 3x - 4y + 5z &= 14 \\ x + y - 8z &= -5 \\ 2x + y + z &= 7 \end{aligned} \quad \longrightarrow \quad A = \left[ \begin{array}{ccc|c} 3 & -4 & 5 & 14 \\ 1 & 1 & -8 & -5 \\ 2 & 1 & 1 & 7 \end{array} \right]$$

- RREF:

$$A' = \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right] \longrightarrow x = 3, y = 0, z = 1$$

# Getting RREF in SageMath

- Create a matrix ← what are the first two arguments of *matrix(.)*?
- Invoke *rref(.)* function



```
A=matrix(3, 4, [3, -4, 5, 14, 1, 1, -8, -5, 2, 1, 1, 7])  
print A.rref()
```

evaluate

```
[1 0 0 3]  
[0 1 0 0]  
[0 0 1 1]
```

# Practice, a Tricky Question

- Try to solve the following equation system using SageMath

$$2x - 5z + y = 6 + w$$

$$5 + z - y = 0$$

$$w + 3(x + y) = z$$

$$1 + 2x - y = w - 3x$$

- Answer:

$$w = \frac{-107}{7}, x = \frac{-12}{7}, y = \frac{54}{7}, z = \frac{19}{7}$$

# Initializing Matrices

- Two ways to create matrices



```
B = matrix(2, 3, [1, 2, 3, 4, 5, 6])  
print B
```

```
[1 2 3]  
[4 5 6]
```



```
B = matrix([ [1, 2, 3], [4, 5, 6] ])  
print B
```

```
[1 2 3]  
[4 5 6]
```

# Exceptional RREF

$$\begin{array}{l} x + 2y + 3z = 7 \\ 4x + 5y + 6z = 16 \\ 7x + 8y + 9z = 24 \end{array} \quad \longrightarrow \quad C1 = \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 4 & 5 & 6 & 16 \\ 7 & 8 & 9 & 24 \end{array} \right]$$



```
B = matrix(3, 4, [1, 2, 3, 7, 4, 5, 6, 16, 7, 8, 9, 24])  
print B.rref()
```

```
[ 1  0 -1  0]  
[ 0  1  2  0]  
[ 0  0  0  1]
```

- What is the solution?

# Exceptional RREF (cont.)

$$\begin{array}{l} x + 2y + 3z = 7 \\ 4x + 5y + 6z = 16 \\ 7x + 8y + 9z = 25 \end{array} \quad \longrightarrow \quad C2 = \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 4 & 5 & 6 & 16 \\ 7 & 8 & 9 & 25 \end{array} \right]$$



```
B = matrix(3, 4, [1, 2, 3, 7, 4, 5, 6, 16, 7, 8, 9, 25])
print B.rref()
```

```
[ 1  0 -1 -1]
[ 0  1  2  4]
[ 0  0  0  0]
```

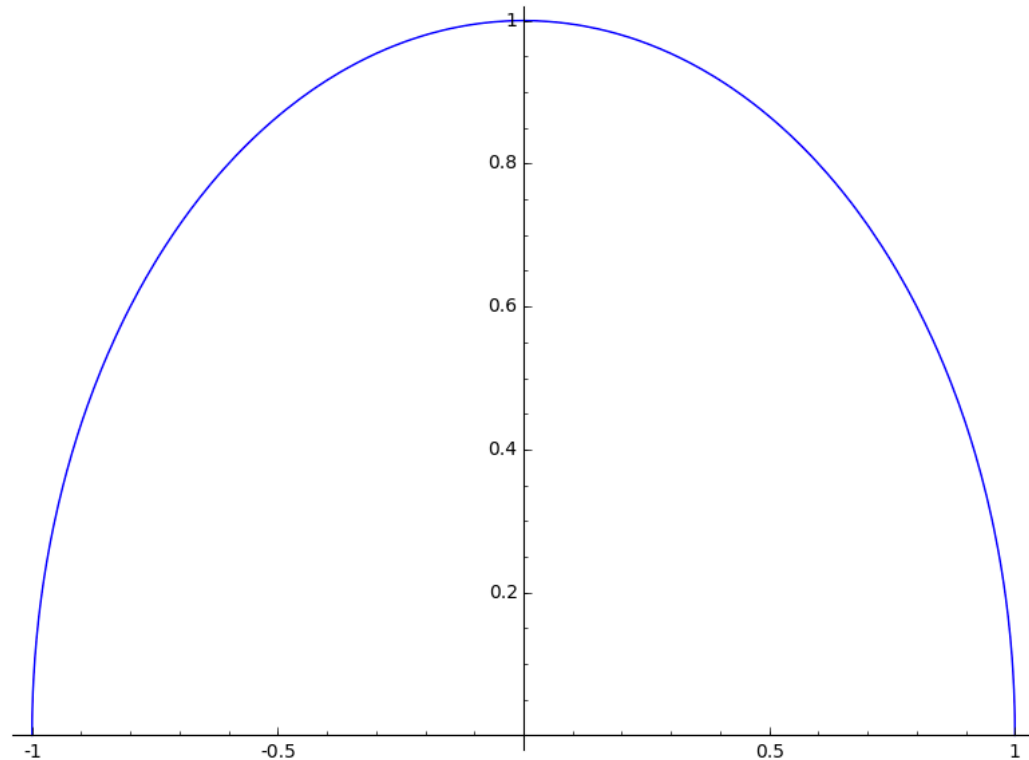
- What are the solutions?

# Define our Own Functions



```
f(x)=sqrt(1-x^2)
print(f(0.1))
print(f(0.001))
print(f(0.00001))
print(f(0.0000001))
print(f(0.000000001))
plot(f, -1, 1)
```

```
0.994987437106620
0.9999999499999875
0.99999999999950000
0.99999999999999995
1.00000000000000000
```



# Function Composition



```
f(x) = 3*x+5
g(x) = x^3 + 1
h(x) = f(g(x))
print h
h(-2)
```

```
x | --> 3*x^3 + 8
-16
```



```
f(x) = 3*x+5
g(x) = x^3 + 1
h(x) = f(g(x))
print h
h2(x) = g(f(x))
print h2
```

```
x | --> 3*x^3 + 8
x | --> (3*x + 5)^3 + 1
```



# Factorization



```
print factor(12345)
print factor(987654321)
print factor(x^2 - 8*x + 15)
print gcd(x^2 - 5*x + 6, x^2 - 8*x + 15)
```

```
3 * 5 * 823
3^2 * 17^2 * 379721
(x - 3)*(x - 5)
x - 3
```

# Expanding a Function



```
a(x) = x^2 - 5 *x + 6
b(x) = x^2 - 8 *x + 15
f(x) = a(b(x))
print f(x)
print f.expand()
print f.factor()
```

```
(x^2 - 8*x + 15)^2 - 5*x^2 + 40*x - 69
x |--> x^4 - 16*x^3 + 89*x^2 - 200*x + 156
(x^2 - 8*x + 13)*(x - 2)*(x - 6)
```

# Use Sage to Solve Problem Symbolically

---

- Solving  $\frac{x^2}{2} - x - 2 = 0$

- Symbolically, we get

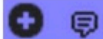
$$1 \pm \sqrt{5}$$

 We will start from here

- Numerically, we get

3.23606797749979 and  $-1.23606797749979$

# Single-Variable Formulas



```
solve(x^2 + 3*x + 2, x)
```

```
[x == -2, x == -1]
```



```
solve(x^2 + 2*x + 3, x)
```

```
[x == -I*sqrt(2) - 1, x == I*sqrt(2) - 1]
```



```
var('alpha')  
solve(cos(alpha) == 1/2, alpha)
```

```
[alpha == 1/3*pi]
```



**Declare variable, notice that x is always a variable without explicit declaration**

# Multiple-Variable Formulas



```
var('a b c')  
solve(a*x^2 + b*x + c == 0, x)
```

```
[x == -1/2*(b + sqrt(b^2 - 4*a*c))/a, x == -1/2*(b - sqrt(b^2 -  
4*a*c))/a]
```



```
var('b')  
solve([x+b ==6, x-b == 4], x, b)
```

```
[[x == 5, b == 1]]
```



```
var('a, b, c')  
solve([9*a + 3*b + c == 32, 4*a + 2*b + c == 15, a + b + c ==6], a, b, c)
```

```
[[a == 4, b == -3, c == 5]]
```



**What does it do? Remember how to solve the same problem using RREF?**

# Non-Linear Equation Systems



```
solve ((x-1) * (x-2) * (x-3) * (x-4) * (x-5) == 0, x)
```

```
[x == 1, x == 2, x == 3, x == 4, x == 5]
```



```
answers = solve ((x-1) * (x-2) * (x-3) * (x-4) * (x-5) == 0, x)
print answers[0]
print answers[3]
```

```
x == 1
x == 4
```



```
var('p q y')
eq1 = p+q == 9
eq2 = q*y + p*x == -6
eq3 = q*y^2 + p*x^2 == 24
eq4 = p == 2
solve([eq1, eq2, eq3, eq4], p, q, x, y)
```

List of lists? How to make it easier to read?



evaluate

```
[[p == 2, q == 7, x == -1/3*sqrt(70) - 2/3, y ==
2/21*sqrt(7)*sqrt(5)*sqrt(2) - 2/3], [p == 2, q == 7, x == 1/3*sqrt(70)
- 2/3, y == -2/21*sqrt(7)*sqrt(5)*sqrt(2) - 2/3]]
```

# Higher Order Equations



```
answers = solve([x^2 - y^2 == 1, (x^2)/4 + (y^2)/3 == 1], x, y)
print answers[0]
print answers[1]
print answers[2]
print answers[3]
```

```
[x == -4/7*sqrt(7), y == -3/7*sqrt(7)]
[x == -4/7*sqrt(7), y == 3/7*sqrt(7)]
[x == 4/7*sqrt(7), y == -3/7*sqrt(7)]
[x == 4/7*sqrt(7), y == 3/7*sqrt(7)]
```



```
solve(log( x^2 ) == 5/3, x)
```

evaluate

**BTW, general polynomial of degree 5 or higher have "no closed-form formula"**



```
solve(sin(x+y)==0.5, x)
```

**Try this to see what we get**

```
[x == 1/6*pi - y]
```

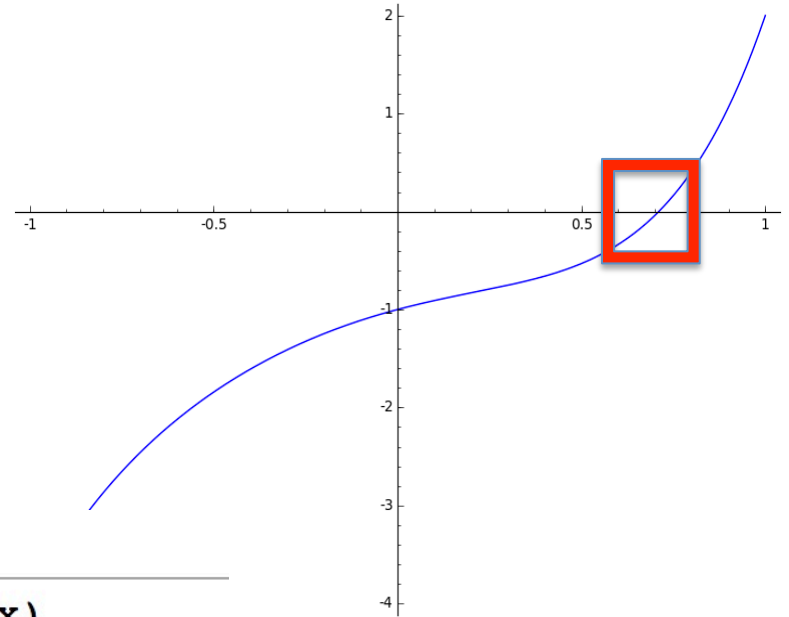


```
var('a0, a1, a2, a3, a4')
solve(a4*x^4 + a3*x^3 + a2*x^2 + a1*x + a0 == 0, x)
```



# Quantic Polynomials and Numerical Solver

$$\text{Solve } x^5 + x^4 + x^3 - x^2 + x - 1 = 0$$



```
solve(x^5 + x^4 + x^3 - x^2 + x - 1 == 0, x)
```

```
[0 == x^5 + x^4 + x^3 - x^2 + x - 1]
```



Why can't we find a solution?



```
find_root(x^5 + x^4 + x^3 - x^2 + x - 1, -100, 100)
```

```
0.7104342557869131
```



# More Numerical Solver Examples



```
find_root(e^x - 1/x, -1, 1)
```

0.5671432904097954



```
find_root(cos(x)==sin(x), 0, pi/2)
```

evaluate

0.7853981633974484



```
var('n')  
find_root(7000 == 5000*(1+0.045/12)^n, 0, 100000)
```

89.89406093308004



**Exercise: what are we calculating?**

# How to Take Derivatives



```
diff(x^4 - x^2 + x , x)
```

$4*x^3 - 2*x + 1$



```
diff(sin(x/e), x)
```

$\cos(x*e^{-1})*e^{-1}$



```
g(x)=e^(-10*x)  
g.derivative()  
f(x) = g.derivative()  
f(-12)
```

$-10*e^{120}$

# Higher Order Derivatives



```
diff(x^3 + 2*x^2 + 3, 2)
```

$6*x + 4$



```
derivative(x^3 + 2*x^2 + 3, 2)
```

$6*x + 4$



```
diff(x^3 + 2*x^2 + 3, 4)
```

$0$

# Indefinite Integral



```
integral(x*cos(x^2), x)
```

$$\int x \cos(x^2) dx = \frac{1}{2} \sin(x^2) ?$$

```
1/2*sin(x^2)
```



```
integral(x / (x^2 + 2), x)
```

$$\int \frac{x}{x^2 + 2} dx = \frac{1}{2} \log(x^2 + 2) ?$$

```
1/2*log(x^2 + 2)
```

**Don't forget the constant!**

# Definite and Impossible Integrals



```
integral(x*cos(x^2), x, -2, 3)
```

```
1/2*sin(9) - 1/2*sin(4)
```

$$\int_{-2}^3 x \cos(x^2) dx$$



```
integral(x / (x^2 + 2), x, 1, 2)
```

```
1/2*log(6) - 1/2*log(3)
```

$$\int_1^2 \frac{x}{x^2 + 2} dx$$



```
var('y')
```

```
assume(y>0)
```

```
integral(2 / sqrt(pi) * (e ^ (-x^2)), x, 0, y)
```

$$\int_0^y \frac{2}{\sqrt{\pi}} e^{-x^2} dx$$



**We even define a special function for this!**

# Numerical Integrations



```
integral(exp(-x^3) * sin(x^2), x, 1, 3)
```

```
integrate(e^(-x^3)*sin(x^2), x, 1, 3)
```



```
numerical_integral(x / (x^2 + 2), 1, 2)
```

```
(0.34657359027997264, 3.84773979655831e-15)
```

# Summary

---

- We introduced SageMath, an opensource project based on Python
- We go over basics of SageMath, including symbolic and numerical solutions, matrices, and simple plots
- We will mostly use SageMath for **symbolic solutions**
  - SageMath was initially designed for this
  - Other tasks are done in Matlab/Octave
- References:
  - <http://www.sagemath.org> ← Official Web and resources
  - <http://www.gregorybard.com/SAGE.html> ← Our textbook

# SageMath #1 Homework (S1)

1. (1%) Find at least 5 real number solutions of  $e^{-x^2} \sin(16x) = 0$  in  $[-1, 1]$  using SageMath  
– Hint: Plot the curve first!
2. (1%) Run  $A = \text{random\_matrix}(\mathbb{Z}\mathbb{Z}, 3, 4)$  in SageMath. Map the matrix into an equation system (say variables are  $x$ ,  $y$ , and  $z$ ). Write down the equation system (in Latex, please turn in the .tex and .pdf). Solve the equation system using SageMath. Present the solution in terms of  $x$ ,  $y$ , and  $z$ .
3. (1%) Execute the following code in SageMath. Explain why the two prints give different answers? You need to identify the key difference to get the point.

```
x = ceil(random() * 888)
print sqrt(x)
print N(sqrt(x))
```



# Preview of Midterm #1



## Midterm #1 (10%)

CS3330 Scientific Computing, Instructor: Cheng-Hsin Hsu

Department of Computing Science, National Tsing Hua University, Taiwan

1:20 p.m. – 3:10 p.m., Oct. 21th, 2016

- **Please create a new latex document, write your solution (no need to copy the questions, but please clearly mark the question numbers in order) into it, typeset it, and submit both your .tex and .pdf files before you leave the classroom. No partial credits will be given to students who fail to submit his/her .pdf file.**
- **You are allowed (actually encouraged) to search online for tips.**
- **You are not allowed to copy and paste source codes from the Internet. Furthermore, you cannot exchange (online/offline) messages with any of your peers during the exam. These are considered as academic dishonesty, which automatically leads to zero point. Furthermore, we will have no choice but report this incident to the university.**

1) (2%) Please reproduce the following formula in Latex  
CS3330 Scientific Computing