SageMath 3: Calculus

Cheng-Hsin Hsu

National Tsing Hua University

Department of Computer Science

Symbolic Calculus

- Online resources to learn more SageMath commands
  - Sage Calculus Tutorial: http://www.sagemath.org/calctut/
  - Sage for Undergraduates: http://www.gregorybard.com/Sage.html
  - Sage Calculus MAA PREP Workshop: http://doc.sagemath.org/html/en/prep/Calculus.html

Find the Materials on Sage Cloud

- All the materials are on the public workspace
  - https://cloud.sagemath.com/projects/1ca56cb9-ebd4-4a01-ba0c-b7b438fecf2c/files/SageMath%203.sagews
- Or download it from
  - https://nmsl.cs.nthu.edu.tw/images/courses/CS3330_2016/SageMath3.zip

Basic Symbolic Computing

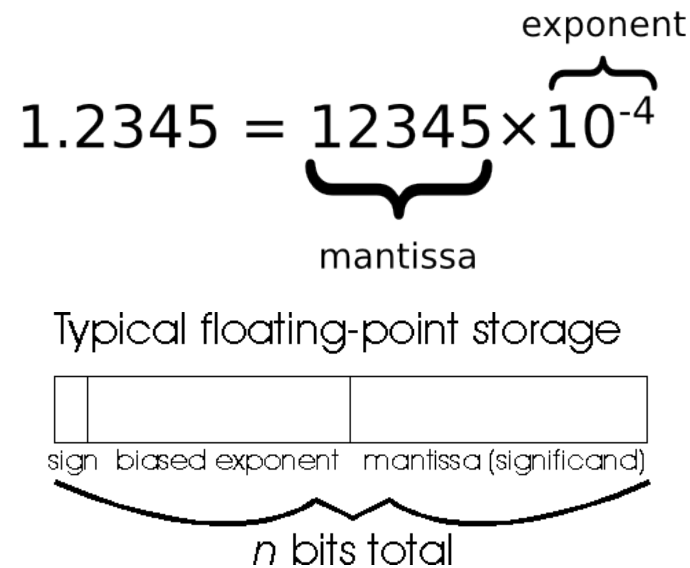
- Defining symbolic variables
- Defining symbolic functions
- Plot
- Differentiate
- Integrate
- Finding zeros

Finding Roots

- Finding roots: symbolic, numerical
- Numerical approximation of a symbolic expression
- More about 2D plotting
- More about 3D plots

How SageMath Stores Floating Point Numbers?

- Try this at home
  - x=100
  - x.digits(base=2)
  - x=100.0
  - x.digits(base=2)



- Precision *p* floating numbers are stored as $sm2^{e-p}$, where $s \in \{-1,1\}, 2^{p-1} \le m < 2^p, -2^{30}+1 \le e \le 2^{30}-1$
  - 0, infinity, -infinity, and NaN are stored differently

More on Integral

- Rounding errors
- Integration assumptions
- Sympy

Midpoint Rule

n 4

f x^2

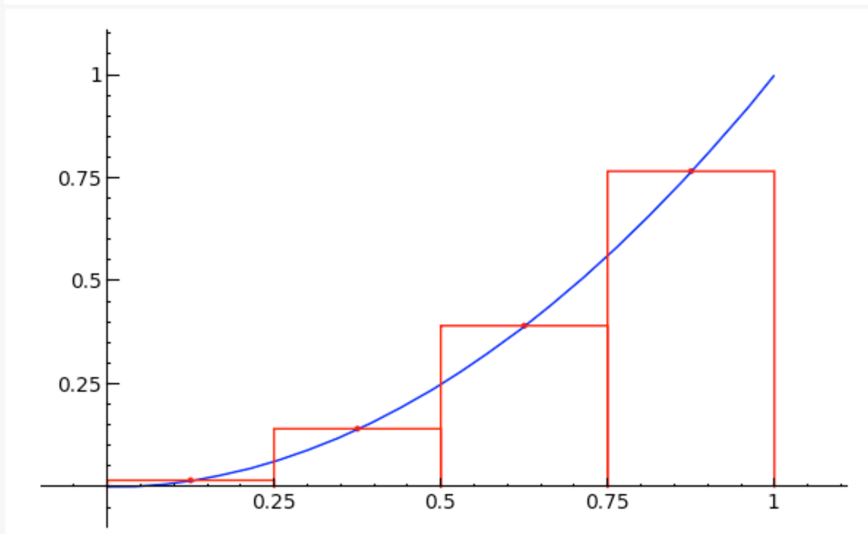
start 0

end 1

Numerical integrals with the midpoint rule

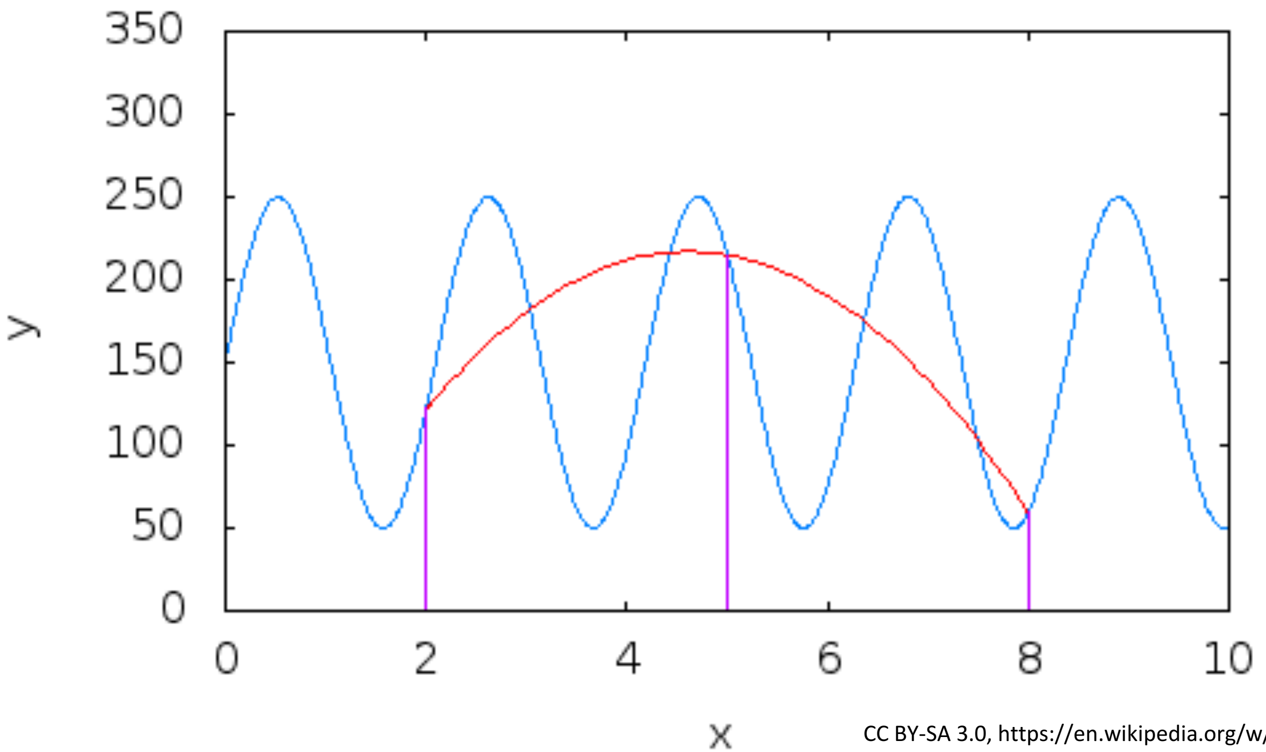
$\int_a^b f(x)dx \approx \sum_i f(x_i)\Delta x$

Sage numerical answer: 0.333333333333
Midpoint estimated answer: 0.328125



Simpson's Rule

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$



SageMath #3 Homework (S3)

1. (1%) Determine the area of the region enclosed between the two curves: $f(x) = -2x(x^2 - 4x + 2)$, $g(x) = -x^2$
2. (1%) What is the derivative of $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$
   Hint: check Piecewise(.) function
3. (1% + 1% Bonus) Simpson's rule and midpoint approximation are two basic numerical integration approaches. Write two SageMath functions to implement them with configurable equally-spaced intervals. You get one bonus point if you compare the accuracy of these two approaches, assuming the numerical approximation of SageMath gives the precise answers.

Please submit your report with all details and source code via the iLMS.
You need to show your work. Just giving an answer gives you 0 point.