```latex
\documentclass{article}
\usepackage{amsthm}
\usepackage{amssymb}
\begin{document}
\newtheorem{thm}{Theorem}
\begin{thm}
  $1^2+3^2+5^2+\cdots+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}.$
\end{thm}
\begin{proof}
  Base case $n=1$.
  \\$1^2=\frac{1(2\times1-1)(2\times1+1)}{3}.$
  \\Suppose the theorem holds when $n=k$.
  \\$1^2+3^2+5^2+\cdots+(2k-1)^2=\frac{k(2k-1)(2k+1)}{3}.$
  \\Let $n=k+1$.
  \\$1^2+3^2+5^2+\cdots+(2k-1)^2+(2k+1)^2=\frac{k(2k-1)(2k+1)}{3}+(2k+1)^2.$
  \\$=\frac{(2k+1)[k(2k-1)+3*(2k+1)]}{3}.$
  \\$=\frac{(2k+1)(2k^2-k+6k+3)}{3}.$
  \\$=\frac{(2k+1)(k+1)(2k+3)}{3}.$
  \\$=\frac{(k+1)[2(k+1)-1][2(k+1)+1]}{3}.$
  \\By the principle of mathematical induction, the theorem holds for all $n \in \mathbb{N}$.
\end{proof}
\end{document}
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Theorem 1. $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$.

Proof. Base case $n = 1$.
$1^2 = \frac{1(2\times1-1)(2\times1+1)}{3}$.
Suppose the theorem holds when $n = k$.
$1^2 + 3^2 + 5^2 + \cdots + (2k-1)^2 = \frac{k(2k-1)(2k+1)}{3}$.
Let $n = k + 1$.
$1^2 + 3^2 + 5^2 + \cdots + (2k-1)^2 + (2k+1)^2 = \frac{k(2k-1)(2k+1)}{3} + (2k+1)^2$.
$= \frac{(2k+1)[k(2k-1)+3*(2k+1)]}{3}$.
$= \frac{(2k+1)(2k^2-k+6k+3)}{3}$.
$= \frac{(2k+1)(k+1)(2k+3)}{3}$.
$= \frac{(k+1)[2(k+1)-1][2(k+1)+1]}{3}$.
By the principle of mathematical induction, the theorem holds for all $n \in \mathbb{N}$. □