Sample Solutions of Programming assignment of Chapter 5: Nonlinear Equations

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Note that, the solutions are for your reference only. If you have any doubts about the correctness of the answers, please let the instructor and the TA know. More importantly, like other math questions, the homework questions may be solved in various ways. Do not assume that the sample solutions here are the only *correct* answers; discuss with others about alternate solutions.

We will not grade your homework assignment, but you are highly encouraged to discuss with us during the Lab hours. The correlation between the homework assignments and quiz/midterm/final questions is high. So you do want to practice more and sooner.

1 Computer Problem

- **5.7**

(a)

```
function cp05_07 % failure of Newton's method
fs = 'x^5-x^3-4*x'; dfs = '5*x^4-3*x^2-4';
x0 = 1; tol = 1e-10; maxits = 10;
disp('(a)␣Newton␣iterates␣alternate␣between␣1␣and␣-1:');
fprintf('\nf(x)␣=␣%s␣=␣0\n', fs); f = inline(fs,'x');
disp('␣k␣x␣f(x)');
k = 0; x = x0; fx = f(x); df = inline(dfs,'x'); delx = 1;
fprintf('%3d␣%17.10e␣%17.10e\n', k, x, fx);
while abs(delx) > tol & k < maxits
       k = k+1; d = df(x); delx = -fx/d; x = x+delx; fx = f(x);
       fprintf('%3d␣%17.10e␣%17.10e\n', k, x, fx);
end; disp('␣');
disp(['(b)␣Roots␣of␣', fs, '␣are:']); roots([1 0 -1 0 -4 0]), disp(
   '␣');
disp('Roots␣are␣simple,␣but␣symmetry␣of␣problems␣causes␣Newton''s␣
   method␣to␣fail.');
fplot(f, [-2 2]); hold on; plot([-1 1], [4 -4], 'x');
```

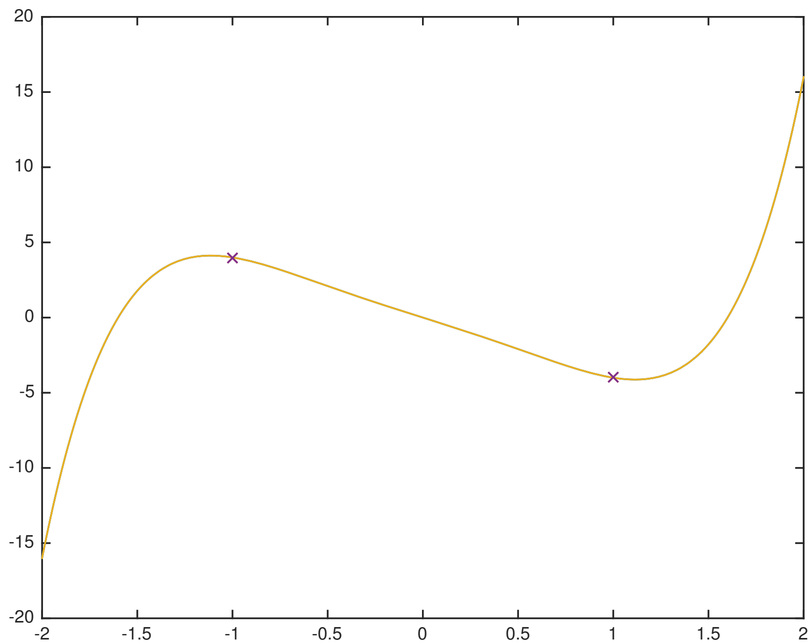

Figure 1: The roots we found in $f(x)$

Result:
$f(x) = x^5 - x^3 - 4x = 0$
Newton iterates alternate between 1 and -1:

k	x	f(x)
0	1.0000000000e+00	-4.0000000000e+00
1	-1.0000000000e+00	4.0000000000e+00
2	1.0000000000e+00	-4.0000000000e+00
3	-1.0000000000e+00	4.0000000000e+00
4	1.0000000000e+00	-4.0000000000e+00
5	-1.0000000000e+00	4.0000000000e+00
6	1.0000000000e+00	-4.0000000000e+00
7	-1.0000000000e+00	4.0000000000e+00
8	1.0000000000e+00	-4.0000000000e+00
9	-1.0000000000e+00	4.0000000000e+00
10	1.0000000000e+00	-4.0000000000e+00

We plotted the roots we found in $f(x)$ in Figure 1.

(b) Roots of $x^5 - x^3 - 4x$ are: $\begin{cases} 0.0000 + 0.0000i \\ 1.6005 + 0.0000i \\ -1.6005 + 0.0000i \\ 0.0000 + 1.2496i \\ 0.0000 - 1.2496i \end{cases}$

- **5.26**

```
function cp05_26 % propane combustion model
R = 4.056734; x1 = 3; x2 = 0; x3 = 2*R; x4 = 0; x5 = 0; x7 = 0; x6 = 8-
    x7;
x9 = 0; x10 = 15+3*R-x7; x8 = R+4-x7-x9-2*x10; % x7 and x9 are
    arbitrary
x0 = [x1; x2; x3; x4; x5; x6; x7; x8; x9; x10];
x = fsolve(@f, x0, optimset('Display','off')), norm_f = norm(f(x))
function [y] = f(x); R = 4.056734; S = sum(x);
y = [x(1)+x(4)-3; 2*x(1)+x(2)+x(4)+x(7)+x(8)+x(9)+2*x(10)-R-10; ...
2*x(2)+2*x(5)+x(6)+x(7)-8; 2*x(3)+x(5)-4*R; x(1)*x(5)-0.193*x(2)*x(4);
    ...
x(6)*sqrt(abs(x(2)))-0.002597*sqrt(abs(x(2)*x(4)*S)); ...
x(7)*sqrt(abs(x(4)))-0.003448*sqrt(abs(x(1)*x(4)*S)); ...
x(4)*x(8)-0.00001799*x(2)*S; x(4)*x(9)-0.0002155*x(1)*sqrt(abs(x(3)*S))
    ; ...
x(4)^2*(x(10)-0.00003846*S)];
```

Result:

$$\begin{aligned} x_1 &= 3.0000, \\ x_2 &= 0, \\ x_3 &= 8.1135, \\ x_4 &= 0, \\ x_5 &= 0, \\ x_6 &= 8.0000, \\ x_7 &= 0, \\ x_8 &= -46.2837, \\ x_9 &= 0, \\ x_{10} &= 27.1702 \end{aligned}$$