Sample Solutions of Programming assignment of Chapter 3: Linear Least Squares

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Note that, the solutions are for your reference only. If you have any doubts about the correctness of the answers, please let the instructor and the TA know. More importantly, like other math questions, the homework questions may be solved in various ways. Do not assume that the sample solutions here are the only *correct* answers; discuss with others about alternate solutions.

We will not grade your homework assignment, but you are highly encouraged to discuss with us during the Lab hours. The correlation between the homework assignments and quiz/midterm/final questions is high. So you do want to practice more and sooner.

1 Computer Problem

- **3.5**

```
function cp03_05 % least squares fit to planetary orbit data
x = [1.02; 0.95; 0.87; 0.77; 0.67; 0.56; 0.44; 0.30; 0.16; 0.01];
y = [0.39; 0.32; 0.27; 0.22; 0.18; 0.15; 0.13; 0.12; 0.13; 0.15];
A = [y.^2 x.*y x y ones(size(x))]; disp('(a)'); b = x.^2; parameters =
   A\b;
problem_ab(x, y, A, 0 , 0, 0, 'Computer␣Problem␣3.5(a)␣-␣Elliptical␣
   Orbit', 1);

disp('(b)');
x2 = x+(rand(size(x))*0.01-0.005); y2 = y+(rand(size(y))*0.01-0.005);
A2 = [y2.^2 x2.*y2 x2 y2 ones(size(x2))]; b2 = x2.^2; parameters2 = A2\
   b2;
problem_ab(x, y, A, 1, x2, A2, 'Computer␣Problem␣3.5(b)␣-␣Perturbed␣
   Orbit', 2);

disp('(c)') %disp('(c) See parts (d)-(f) below.');
rk1 = zeros(1,5); rk2 = zeros(1,5); tol_parameters = zeros(5,5);
for tol=1:5
       rk1(tol) = rank(A, 10^(-tol));
       rk2(tol) = rank(A2, 10^(-tol));
       A_tol = A; A2_tol = A2;
       A_tol(A < 10^(-tol)) = 0; %tolerance
```

```
       A2_tol(A2 < 10^(-tol)) = 0; %tolerance
       fig_title = strcat('Computer Problem 3.5(c) - Perturbed Orbit - 
          tol k = ',...
       int2str(tol), ' ; rank(A) = ', int2str(rk1(tol)), ' ; rank(A2) = 
          ', int2str(rk2(tol)));
       tol_parameters(:,tol) = problem_ab(x, y, A_tol, 1, x2, A2_tol, 
          fig_title, tol+2);
end
disp('tolerance 1 : 1 2 3 4 5');
fprintf('rank(A): : %d %d %d %d %d\n\n', rk1);
fprintf('rank(A2): : %d %d %d %d %d\n\n', rk2);

disp('(d)'); [U, S, V] = svd(A);

disp('(e)'); figure(8);
parameters = zeros(5,1);
for k = 1:5
       parameters = parameters+U(:,k)'*b*V(:,k)/S(k,k)
       [xs, ys] = meshgrid(-1:0.1:2, -1:0.1:2);
       contour(-1:0.1:2, -1:0.1:2, parameters(1)*ys.^2+parameters(2)*xs
          .*ys+ ...
       parameters(3)*xs+parameters(4)*ys+parameters(5)-xs.^2, [0, 0], '
          k-');
       hold on;
end; title('Computer Problem 3.5(e) - Orbits Using SVD'); plot(x, y, '
   bx');
hold off;

disp('(f)'); figure(9);
[U2, S2, V2] = svd(A2); parameters2 = zeros(5,1);
for k = 1:5
       parameters2 = parameters2+U2(:,k)'*b2*V2(:,k)/S2(k,k)
       [xs, ys] = meshgrid(-1:0.1:2, -1:0.1:2);
       contour(-1:0.1:2, -1:0.1:2, parameters2(1)*ys.^2+parameters2(2)*
          xs.*ys+ ...
       parameters2(3)*xs+parameters2(4)*ys+parameters2(5)-xs.^2, [0, 
          0], 'k-');
       hold on;
end; title('Computer Problem 3.5(f) - Perturbed Orbits Using SVD');
   plot(x, y, 'bx');
hold off;
```

```
disp('(g)'); figure(10);
[U, S, V] = svd([A b]); parameters3 = -1/V(6, 6)*V(1:5, 6)
[xs, ys] = meshgrid(-1:0.1:2, -1:0.1:2);
contour(-1:0.1:2, -1:0.1:2, parameters3(1)*ys.^2+parameters3(2)*xs.*ys+
    ...
parameters3(3)*xs+parameters3(4)*ys+parameters3(5)-xs.^2, [0, 0], 'r-')
   ;
hold on;
contour(-1:0.1:2, -1:0.1:2, parameters(1)*ys.^2+parameters(2)*xs.*ys+
   ...
parameters(3)*xs+parameters(4)*ys+parameters(5)-xs.^2, [0, 0], 'k-');
hold on; title('Computer␣Problem␣3.5(g)␣-␣Orbit␣Using␣Total␣Least␣
   Squares');
plot(x, y, 'bx'); hold off;
end

%% problem a, b
function parameters = problem_ab(x, y, A, b_swich, x2, A2, fig_title,
   fig_num)
b = x.^2; parameters = A\b
figure(fig_num);
[xs, ys] = meshgrid(-1:0.1:2, -1:0.1:2);
if b_swich == 1
       b2 = x2.^2; parameters2 = A2\b2
       contour(-1:0.1:2, -1:0.1:2, parameters2(1)*ys.^2+parameters2(2)*
          xs.*ys+ ...
       parameters2(3)*xs+parameters2(4)*ys+parameters2(5)-xs.^2, [0,
          0], 'r-');
       hold on;
end
contour(-1:0.1:2, -1:0.1:2, parameters(1)*ys.^2+parameters(2)*xs.*ys+
   ...
parameters(3)*xs+parameters(4)*ys+parameters(5)-xs.^2, [0, 0], 'k-');
hold on;
title(fig_title); plot(x, y, 'bx');
hold off;
end
```

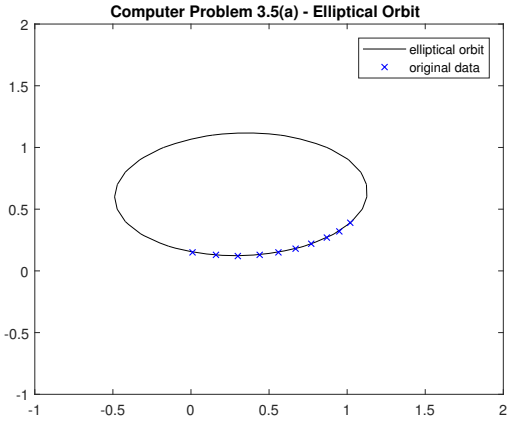

Figure 1: problem(a)

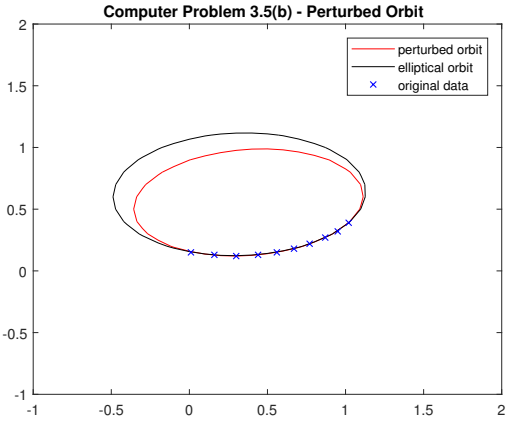

Figure 2: problem(b)

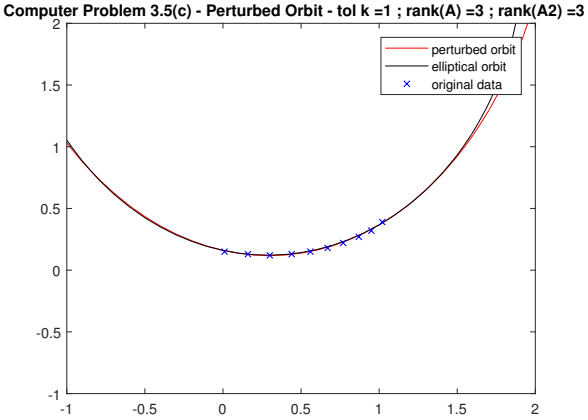

Figure 3: problem(c) k=1

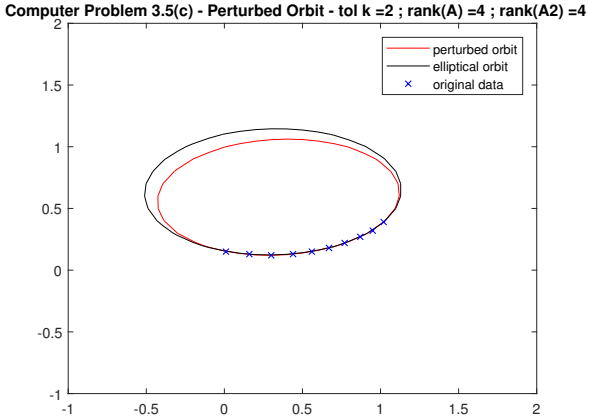

Figure 4: problem(c) k=2

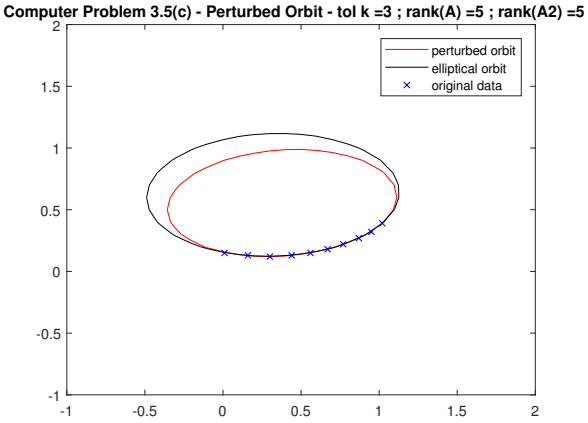

Figure 5: problem(c) k=3

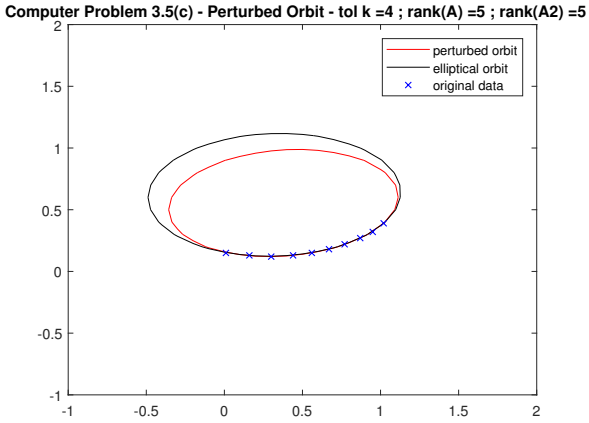

Figure 6: problem(c) k=4

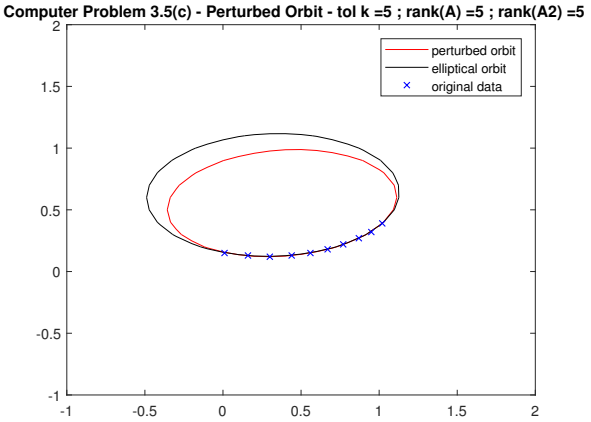

Figure 7: problem(c) k=1

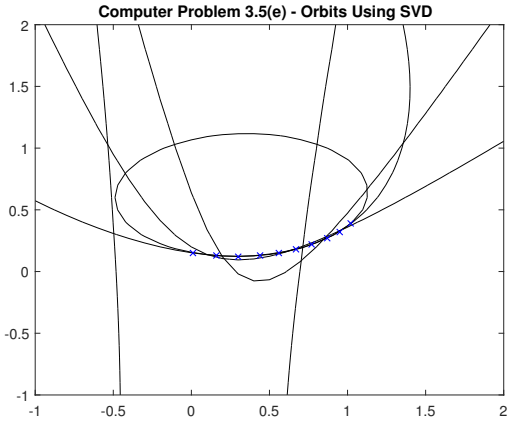

Figure 8: problem(e)

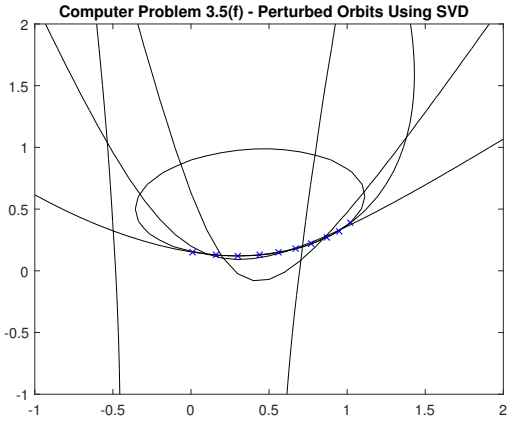

Figure 9: problem(f)

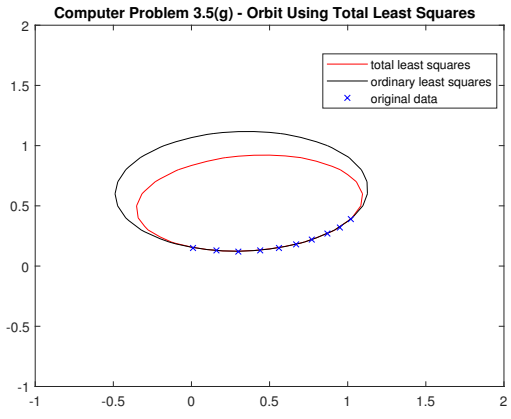

Figure 10: problem(g)

(a) parameters = $\begin{matrix} -2.6356 \\ 0.1436 \\ 0.5514 \\ 3.2229 \\ -0.4329 \end{matrix}$. The results are given in Figure 1

(b) parameters = $\begin{matrix} -2.8528 \\ 0.4144 \\ 0.5258 \\ 3.0051 \\ -0.3982 \end{matrix}$. The results are given in Figure 2

(c)

k	1	2	3	4	5
elliptical rank	3	4	5	5	5
elliptical parameters	-0.4352 -0.0101 0.5783 2.2823 -0.3533	-2.5697 0.1018 0.5574 3.2305 -0.4352	-2.6356 0.1436 0.5514 3.2229 -0.4329	-2.6356 0.1436 0.5514 3.2229 -0.4329	-2.6356 0.1436 0.5514 3.2229 -0.4329
perturbed rank	3	4	5	5	5
perturbed parameters	-0.3051 -0.0316 0.5989 2.1619 -0.3385	-2.6951 0.2547 0.5411 3.0995 -0.4110	-2.8528 0.4144 0.5258 3.0051 -0.3982	-2.8528 0.4144 0.5258 3.0051 -0.3982	-2.8528 0.4144 0.5258 3.0051 -0.3982

The results are given in Figure 3 - 7

(d) $U =$

$$\begin{bmatrix} -0.3918 & -0.4222 & 0.5675 & 0.3703 & -0.3960 & -0.1254 & -0.1449 & -0.0502 & 0.0339 & -0.1090 \\ -0.3750 & -0.3201 & 0.1514 & -0.0960 & 0.4586 & 0.3426 & 0.3799 & 0.2350 & 0.1429 & 0.4208 \\ -0.3588 & -0.2234 & -0.0839 & -0.3191 & 0.3300 & -0.0031 & -0.0432 & -0.2502 & -0.4946 & -0.5426 \\ -0.3401 & -0.1124 & -0.2623 & -0.3350 & -0.0140 & -0.4688 & -0.4310 & -0.1949 & 0.1798 & 0.4632 \\ -0.3226 & -0.0094 & -0.3503 & -0.1603 & -0.2820 & -0.1285 & 0.2065 & 0.5495 & 0.3700 & -0.4122 \\ -0.3046 & 0.0953 & -0.3454 & 0.0885 & -0.3529 & 0.7301 & -0.2623 & -0.1855 & -0.0438 & 0.0911 \\ -0.2862 & 0.2028 & -0.2572 & 0.3286 & -0.1500 & -0.2766 & 0.6369 & -0.3757 & -0.1835 & 0.1525 \\ -0.2658 & 0.3225 & -0.0805 & 0.4674 & 0.2835 & -0.1372 & -0.3276 & 0.4942 & -0.3752 & 0.0940 \\ -0.2466 & 0.4367 & 0.1797 & 0.1614 & 0.3849 & 0.0372 & -0.1129 & -0.3292 & 0.5850 & -0.2780 \\ -0.2264 & 0.5583 & 0.4819 & -0.5036 & -0.2621 & 0.0297 & 0.0986 & 0.1070 & -0.2144 & 0.1202 \end{bmatrix}$$

$$S = \begin{bmatrix} 3.7860 & 0 & 0 & 0 & 0 \\ 0 & 0.9449 & 0 & 0 & 0 \\ 0 & 0 & 0.2089 & 0 & 0 \\ 0 & 0 & 0 & 0.0230 & 0 \\ 0 & 0 & 0 & 0 & 0.0055 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$v = \begin{bmatrix} -0.0464 & -0.0940 & 0.3459 & 0.3250 & -0.8739 \\ -0.1341 & -0.3141 & 0.5899 & 0.5524 & 0.4799 \\ -0.5188 & -0.7484 & -0.4027 & -0.0581 & -0.0729 \\ -0.1805 & -0.1418 & 0.6085 & -0.7595 & -0.0168 \\ -0.8235 & 0.5589 & 0.0048 & 0.0948 & 0.0207 \end{bmatrix}$$

(e)

k	1	2	3	4	5
parameter	0.0190	0.1044	0.4192	-0.6528	-2.6356
	0.0547	0.3401	0.8769	-0.9451	0.1436
	0.2117	0.8918	0.5253	0.7168	0.5514
	0.0737	0.2025	0.7561	3.2610	3.2229
	0.3361	-0.1718	-0.1674	-0.4800	-0.4329

The results are given in Figure 8

(f)

k	1	2	3	4	5
parameter	0.0189	0.1042	0.4101	-0.5829	-2.8528
	0.0547	0.3391	0.8598	-0.8321	0.4144
	0.2122	0.8904	0.5333	0.7153	0.5258
	0.0735	0.2028	0.7467	3.0466	3.0051
	0.3367	-0.1700	-0.1655	-0.4516	-0.3982

The results are given Figure 9

(g) see Figure 10