Worksheet #10 (2017/11/6)

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- Supplementary materials to warm you up for least squares.

1) Try to minimize $f(x,y,z) = \frac{1}{2}\begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} 4 & 1 & 2 \\ 1 & 8 & 5 \\ 2 & 5 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$.

2) Let's look at a very primitive quadratic function $f(x) = \frac{1}{2}ax^2 - bx$. What is its minimum?

3) Revisit $f(x, y, z) = \frac{1}{2} \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 4 & 1 & 2 \\ 1 & 8 & 5 \\ 2 & 5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$. Rewrite it as $\frac{1}{2}\mathbf{x}^T\mathbf{A}\mathbf{x} - \mathbf{x}^T\mathbf{b}$. What is its minimum?

4) **Algebraic view of least squares:** For a very "tall" $\mathbf{Ax} = \mathbf{b}$, we let $\mathbf{r} = \mathbf{b} - \mathbf{Ax}$. Then, we try to minimize $\mathbf{r}^T\mathbf{r}$, which is the sum of square of deviation along the y-axis. Show that $\mathbf{x} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}$.

5) **Geometric view:** Find the α_1 and α_2 for the projection of $\vec{b}$, so that $\alpha_1\vec{a}_1 + \alpha_2\vec{a}_2$ lines on the plane. Connect the dots with the famous formula $\mathbf{x} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}$.