Worksheet #2 (2017/09/18)

Name: ID:

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Note: we will collect this worksheet at the end of the lecture.

- We plan to cover Sections 1.2.5–1.3.7 today.
- We use Chapter 01 slides 13–35.
- This is corresponding to the textbook pages 10–22.

1) Let $f(x) = e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$, assume we implement a $\hat{f}(x) = \sum_{i=0}^{3} \frac{x^i}{i!}$ to approximate e^x. Analyze the backward and forward errors at $x = -1$ and $x = 1$. Use 2.718281828459046 in you calculations. (Feel free use any calculator, dedicated ones, apps on smartphones, or applications on laptops. But remember to show your work.)

2) For function f with approximate input $\hat{x} = x + \Delta x$ instead of true x, we have the condition number of $f(\cdot)$ as $\left|\frac{xf'(x)}{f(x)}\right|$. Let $g(y) = f^{-1}(y)$ and $y = f(x)$, from calculus, we know $g'(y) = \frac{1}{f'(x)}$ (let's assume $f'(x) \neq 0$).
   a) What is the condition number of $g(\cdot)$?
   b) What do you observe?

3) Consider $f(x) = \sqrt{x}$, since $f'(x) = \frac{1}{2\sqrt{x}}$:

   a) Find its condition number.

   b) How to interpret the number? Is this (square root) problem well-conditioned?

4) (Multiple choice) What are the following causes of inaccuracy of solutions produced by your algorithm? (a) ill-conditioned problem, (b) unstable problem, and (c) both?

5) Round the following decimal numbers to two digits using the specified rounding rules.

Number	Chop	Round to Nearest	Number	Chop	Round to Nearest
1.649			1.749		
1.650			1.750		
1.651			1.751		
1.699			1.799		

6) Consider our toy floating point system: $\beta = 2$, $p = 3$, $L = -1$, $U = 1$ with rounding to the nearest, what is its ϵ_{mach} and UFL?