

Worksheet #4 (2017/09/25)

Name:

ID:

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- We plan to cover Sections 2.1–2.3.5 (inclusive) today.
- We use Chapter 02 slides 1–25.
- This is corresponding to the textbook pages 49–63.

1) Solve
$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 26 \\ 34 \\ 39 \end{bmatrix},$$
 by hand and with Matlab.

- 2) Linear equation system finds an unknown n -vector \mathbf{x} for given $m \times n$ matrix \mathbf{A} and m -vector \mathbf{b} to satisfy $\mathbf{Ax} = \mathbf{b}$. What happen if: (i) $m > n$, (ii) $m < n$, and (iii) $m = n$?

3) Solve
$$\begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix},$$
 if $[b_1 \ b_2]^T = [8 \ 13]^T$. Also solve
$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix},$$
 if (i) $[b_1 \ b_2]^T = [4 \ 7]^T$, and (ii) $[b_1 \ b_2]^T = [4 \ 8]^T$.

4) Consider
$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ 3 & -1 & 4 \end{bmatrix},$$
 what is its 1- and ∞ -norm?

- 5) What is the $\text{cond}_1(\mathbf{A})$, and $\text{cond}_\infty(\mathbf{A})$ with \mathbf{A} defined above?

6)
$$\mathbf{A} = \begin{bmatrix} 0.913 & 0.659 \\ 0.457 & 0.330 \end{bmatrix}$$
 and $\mathbf{b} = [0.254 \ 0.127]^T$. Consider two approximate solutions $\hat{x}_1 = [-0.0827 \ 0.5]^T$ and $\hat{x}_2 = [0.999 \ -1.001]^T$. Their norms are $\|\hat{r}_1\|_1 = 2.1 \times 10^{-4}$ and $\|\hat{r}_2\|_1 = 2.4 \times 10^{-2}$. Which solution do we prefer?