Worksheet #5 (2017/10/2)

Name: ID:

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Note: we will collect this worksheet at the end of the lecture.

- We plan to cover Sections 2.4.1–2.4.4 (inclusive) today.
- We use Chapter 02 slides 26–48.
- This is corresponding to the textbook pages 63–70.

1) First make sure you understand what is premultiplying and postmultiplying. If you are still not sure, please ask the instructor to slow down and clarify. Between them postmultiplying is less straightforward, because it *will* change the solution of $\boldsymbol{Ax} = \boldsymbol{b}$. Say we multiple the both side of the linear system with a permutation matrix $\boldsymbol{P}$, and we write the new equation as $\boldsymbol{APx'} = \boldsymbol{b}$. Note that $\boldsymbol{b}$ is not changed, because post multiplying $\boldsymbol{A}$ by $\boldsymbol{P}$ essentially *permutes* some elements in $\boldsymbol{x}$ into $\boldsymbol{x'}$.
   Use the space below to Show that $\boldsymbol{x} = \boldsymbol{Px'}$.

2) Find a permutation matrix $\boldsymbol{P}$ to transform $\boldsymbol{A} = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ into a upper triangular matrix, i.e., $\boldsymbol{PA}$ is a upper triangular matrix.

3) In the lecture, we discuss how to solve $\begin{bmatrix} 2 & 4 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}$ using back-substitution. What we didn't do is to write down the two matrices used in the back substitution; one to eliminate all the 3rd-column elements in the upper triangle; and the other one for the 2nd-column elements. Please write them down in the space below.

4) Please perform LU decomposition of $\boldsymbol{A} = \begin{bmatrix} 1 & 1 \\ & \\ 1 & 1 \end{bmatrix}$. Please show your work and write down $\boldsymbol{L}$ and $\boldsymbol{U}$.