Worksheet #7 (2017/10/18)

Name: ID:

CS3330 Scientific Computing, Instructor: Cheng-Hsin Hsu

- We plan to cover Sections 5.1–5.5.1 (inclusive) today.
- We use Chapter 05 slides 1–17.
- This is corresponding to the textbook pages 216–226.

1) Please write down the general *Intermediate Value* theorem using $f : \mathbb{R} \to \mathbb{R}$. How do we use it to check the existence of solutions (roots) of the nonlinear function f?

2) Another way to check the existence of zeros is to use the *Inverse Function* theorem. This theorem says that for a continuously differentiable function $\mathbf{f}$, if the Jacobian matrix $\mathbf{J}_f$, defined by $\{\mathbf{J}(\mathbf{x})\}_{i,j} = \partial f_i(\mathbf{x})/\partial x_j$ is nonsingular at a point $\mathbf{x}^*$, then there is a neighborhood of $\mathbf{f}(\mathbf{x}^*)$ where $\mathbf{f}$ is invertible. Therefore, we can simply evaluate $\mathbf{f}^{-1}(0)$ to get the root. However, even if $\mathbf{f}$ is locally invertible everywhere, it still may not be globally invertible. Why? Give a counter example.

3) Consider the following nonlinear function system:

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} x_1^2 & -x_2 & +0.25 \\ -x_1 & +x_2^2 & +0.25 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Check if its Jacobian matrix is singular at $\mathbf{x}^* = [0.5, 0.5]^T$. What does it mean?

4) If the following numbers represent the magnitudes of errors at successive iterations, what is the convergence rate of individual iterative algorithms?

a) $10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}$

b) $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}$

c) $10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$

d) $10^{-2}, 10^{-4}, 10^{-8}, 10^{-16}$