Worksheet #8 (2017/10/23)

Name: ID:

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Note: we will collect this worksheet at the end of the lecture.

- We plan to cover Sections 5.5.2–5.5.8 (inclusive) today.
- We use Chapter 05 slides 18–40.
- This is corresponding to the textbook pages 226–237.

1) $x_0, x_1, \ldots, x_k$ is a sequence of approximations from an iterative algorithm. The convergence analysis tells us whether the algorithm *will* converge, but it doesn't tell us when to stop. In other words, our textbook doesn't discuss when to declare the approximation is good enough, and assumes that infinite number of iterations are applied. Therefore, the *practical/suitable* stopping criteria are left to users to determine. Two possible definitions of stopping criteria are: (a) checking if $\parallel x_{k+1} - x_k \parallel / \parallel x_k \parallel$ is small enough or (b) checking if the residue $\parallel f(x_k) \parallel$ is small enough. Would these two values always be small simultaneously?

2) For each $f(x) = 0$, there may be multiple corresponding fixed-point problems $x = g(x)$ with different $g(\cdot)$ functions. Please verify the following problems: (a) $g(x) = x^2 - 2$, (b) $g(x) = \sqrt{x+2}$, (c) $g(x) = 1 + 2/x$, and (d) $g(x) = (x^2+2)/(2x-1)$ are the fixed-point problems of the nonlinear equation $f(x) = x^2 - x - 2 = 0$.

3) Follow the previous question, analyze the convergence rate of the four fixed-point problems.

4) Newton's method can be seen as approximating roots using linear function. What is Muller's method? Even higher-order polynomials can be used, but what are their drawbacks?