Department of Computer Science
National Tsing Hua University

CS 5244: Introduction to Cyber Physical Systems

Unit 13: Synchronous/Reactive Models (Ch. 6)

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Concurrent Composition: Alternatives to Threads

Threads yield incomprehensible behaviors.

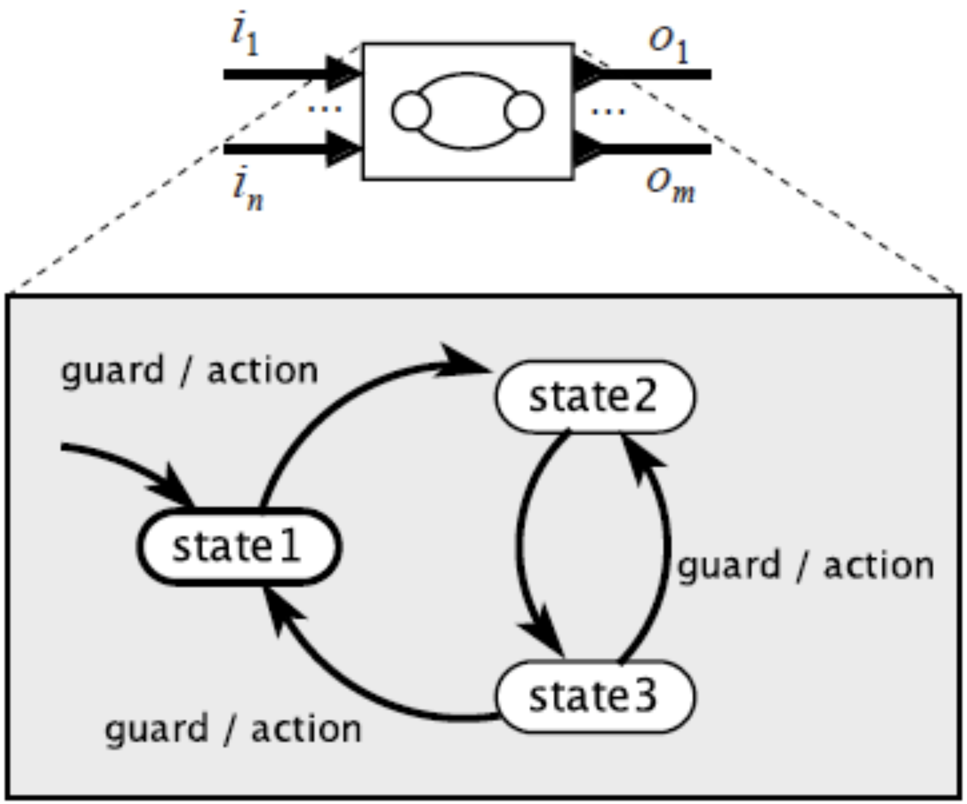
Composition of State Machines:

- Side-by-side composition
- Cascade composition
- Feedback composition

We will begin with synchronous composition, an abstraction that has been very effectively used in hardware design and is gaining popularity in software design.

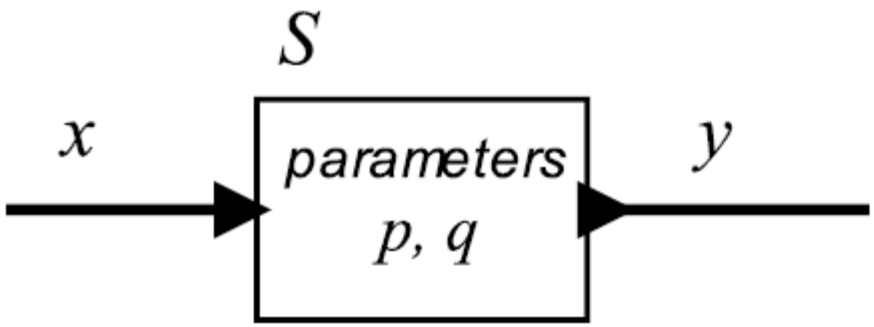
Recall: Actor Model for State Machines

Expose inputs and outputs, enabling composition:



Recall: Actor Model of Continuous-Time Systems

A *system* is a function that accepts an input *signal* and yields an output signal.



The domain and range of the system function are sets of signals, which themselves are functions.

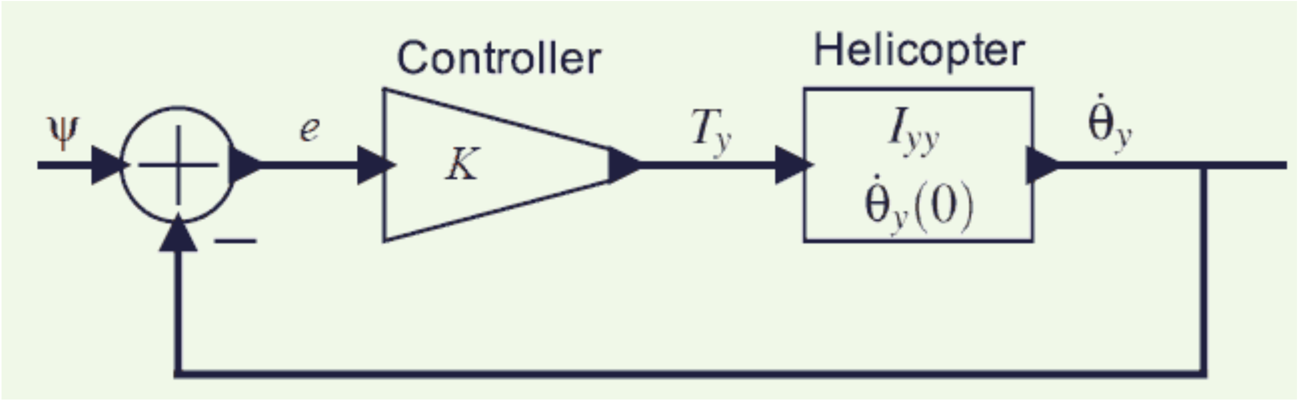
$$x\colon \mathbb{R} \to \mathbb{R}, \quad y\colon \mathbb{R} \to \mathbb{R}$$

$$S\colon X \to Y$$

$$X = Y = (\mathbb{R} \to \mathbb{R})$$

Parameters may affect the definition of the function *S*.

Recall: Composition of Actors

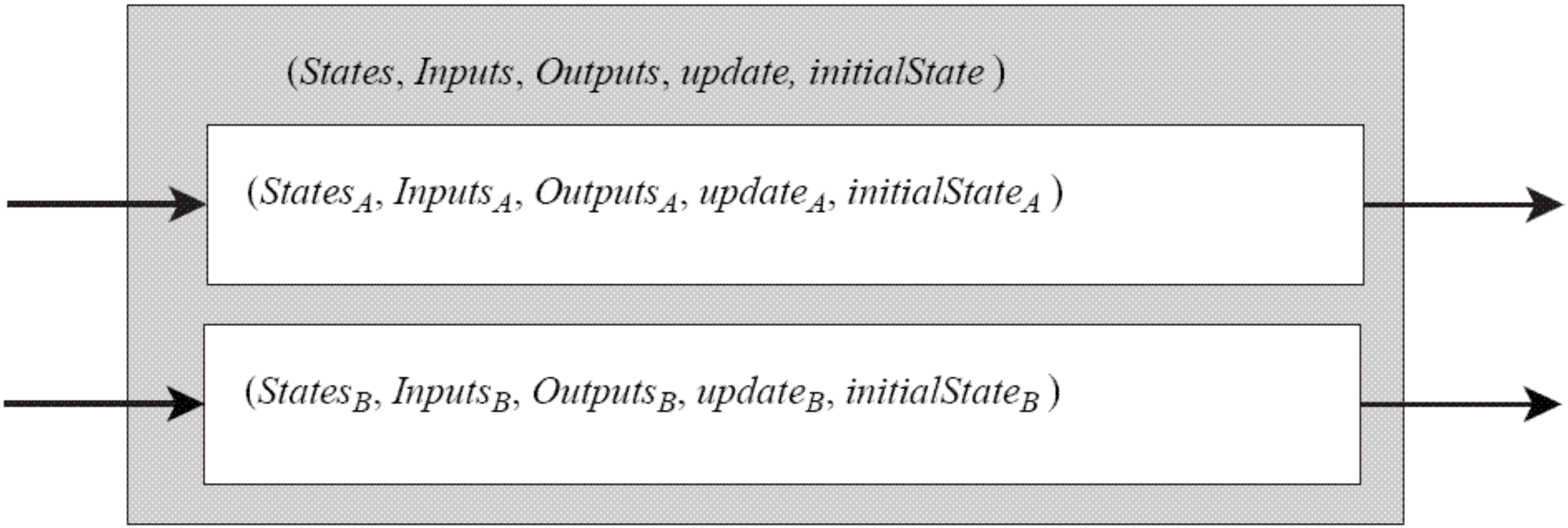


$$\begin{aligned}\dot{\theta}_y(t) &= \dot{\theta}_y(0) + \frac{1}{I_{yy}} \int_0^t T_y(\tau) d\tau \\ &= \dot{\theta}_y(0) + \frac{K}{I_{yy}} \int_0^t (\psi(\tau) - \dot{\theta}_y(\tau)) d\tau\end{aligned}$$

Angular velocity appears on both sides. The *semantics* (meaning) of the model is the solution to this equation.

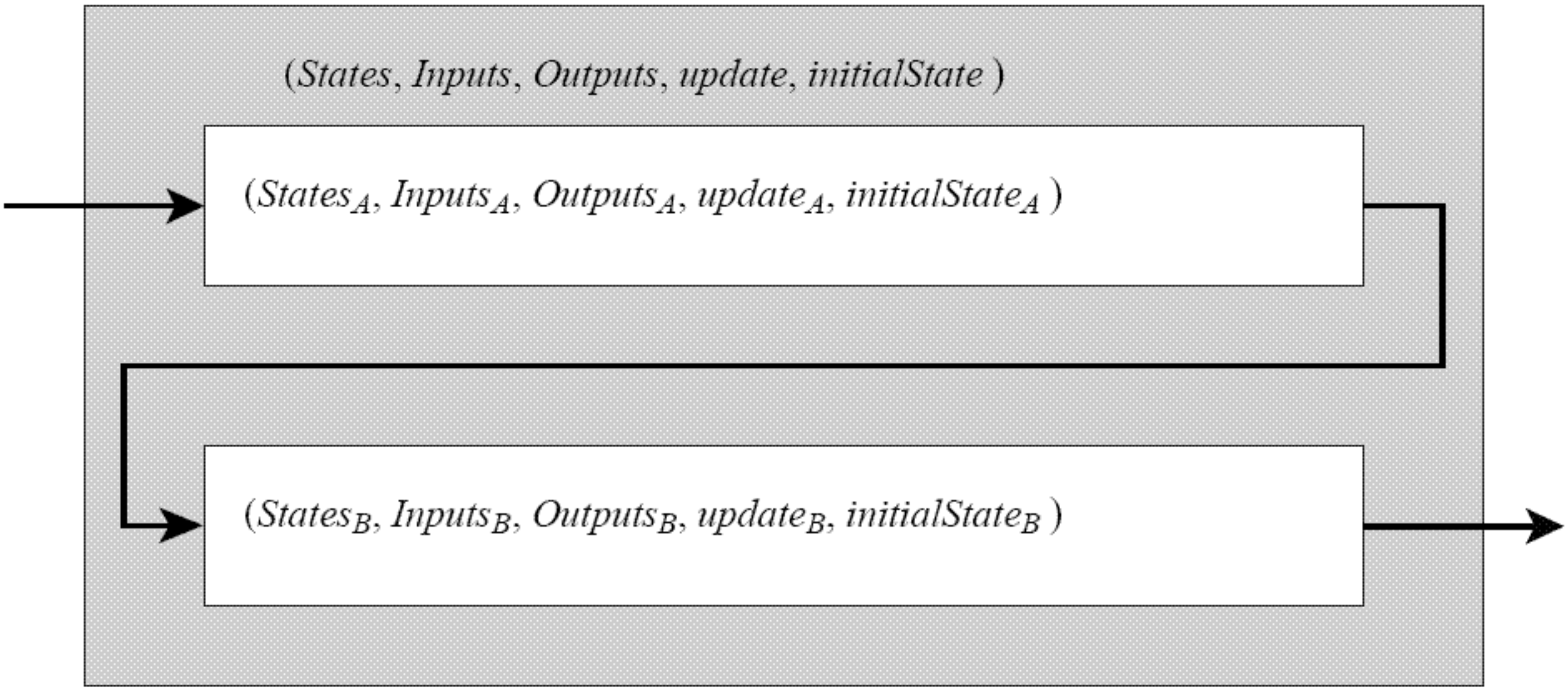
We will now generalize this notion of composition.

Side-by-Side Composition



Synchronous composition: the machines react simultaneously and instantaneously.

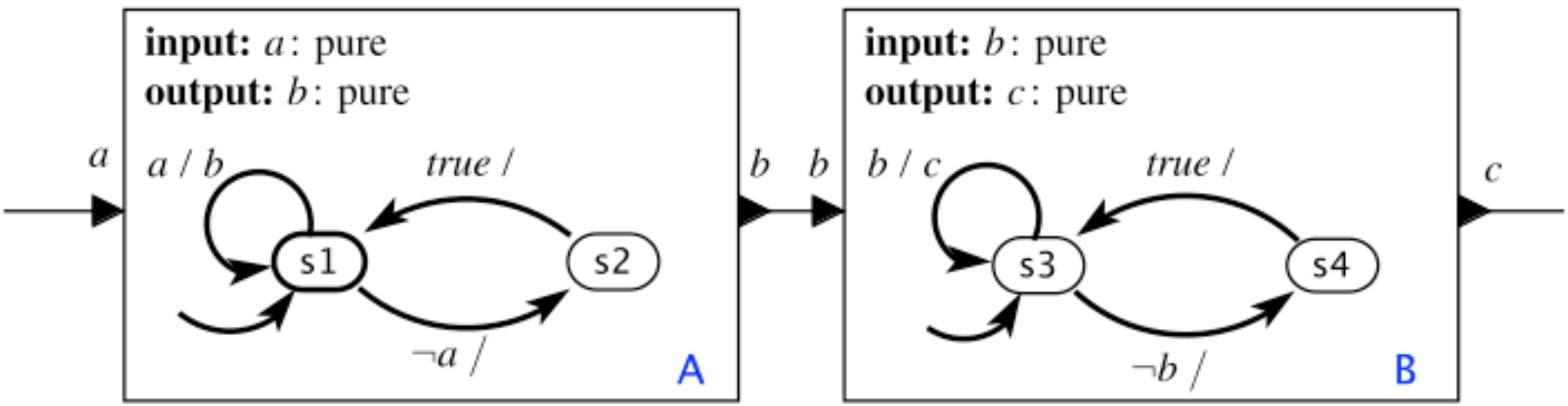
Cascade Composition



Synchronous composition: the machines react simultaneously and instantaneously, despite the apparent causal relationship!

Synchronous Composition: Reactions are *Simultaneous* and *Instantaneous*

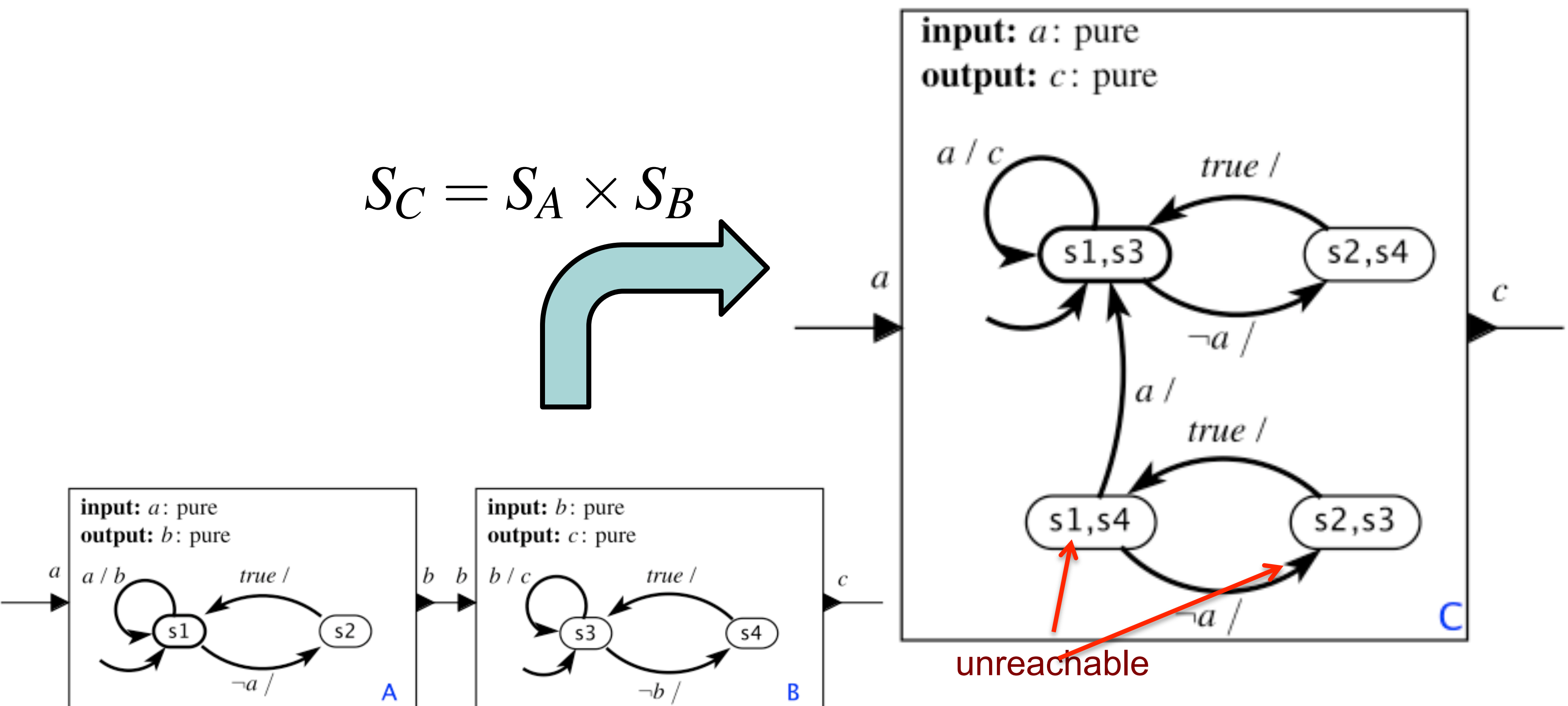
Consider a cascade composition as follows:



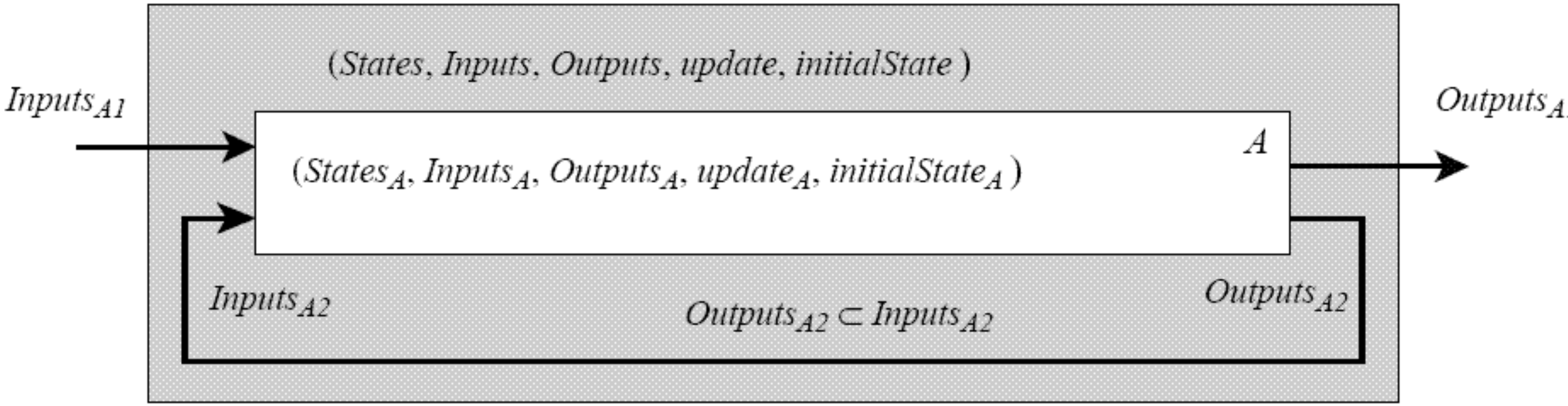
Synchronous Composition:
Reactions are *Simultaneous* and *Instantaneous*

In this model, you must not think of machine A as reacting before machine B. If it did, the unreachable states would not be unreachable.

$$S_C = S_A \times S_B$$

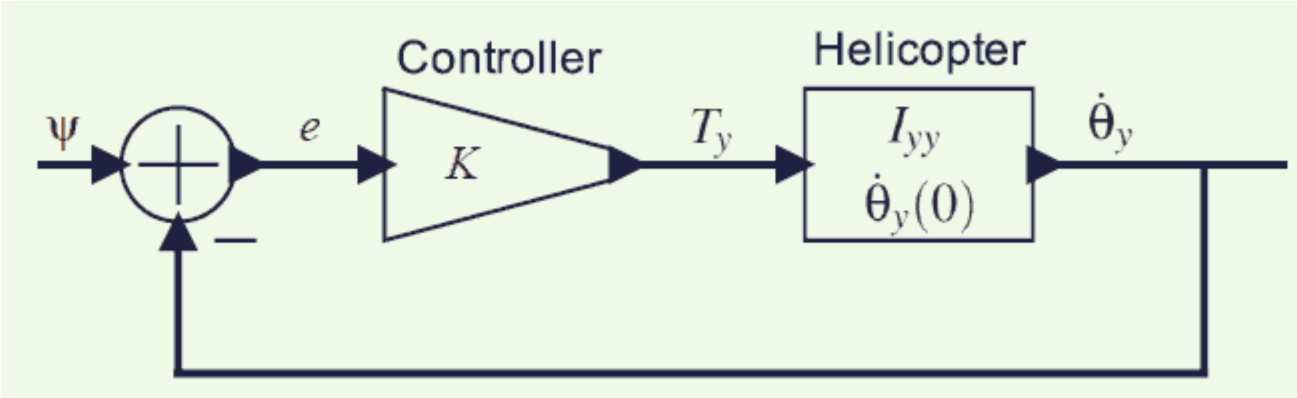


Feedback Composition



Turns out everything can be viewed as feedback composition…

Example: Feedback Composition

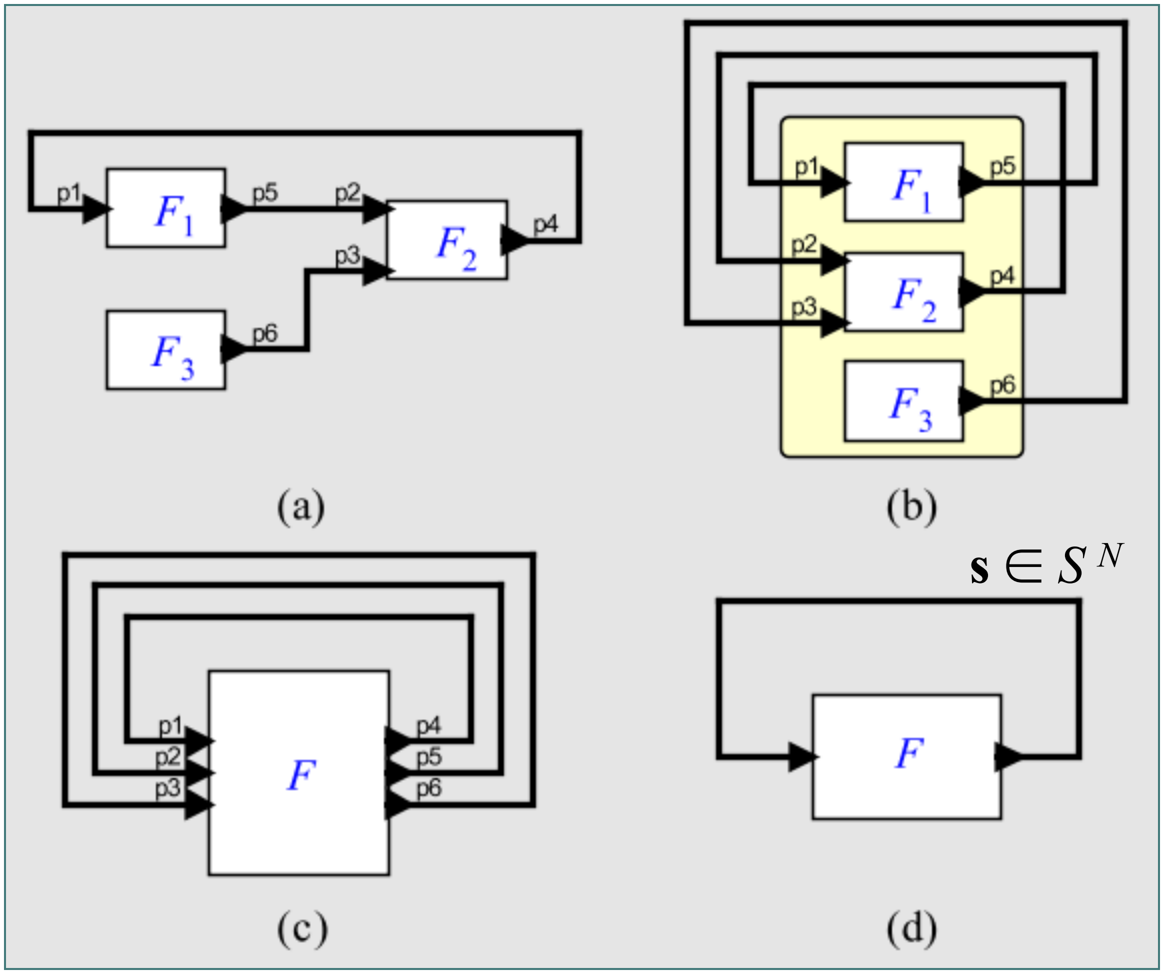


$$\dot{\theta}_y(t) = \dot{\theta}_y(0) + \frac{1}{I_{yy}} \int_0^t T_y(\tau) d\tau$$

$$= \dot{\theta}_y(0) + \frac{K}{I_{yy}} \int_0^t (\psi(\tau) - \dot{\theta}_y(\tau)) d\tau$$

Angular velocity appears on both sides. The semantics (meaning) of the model is the solution to this equation.

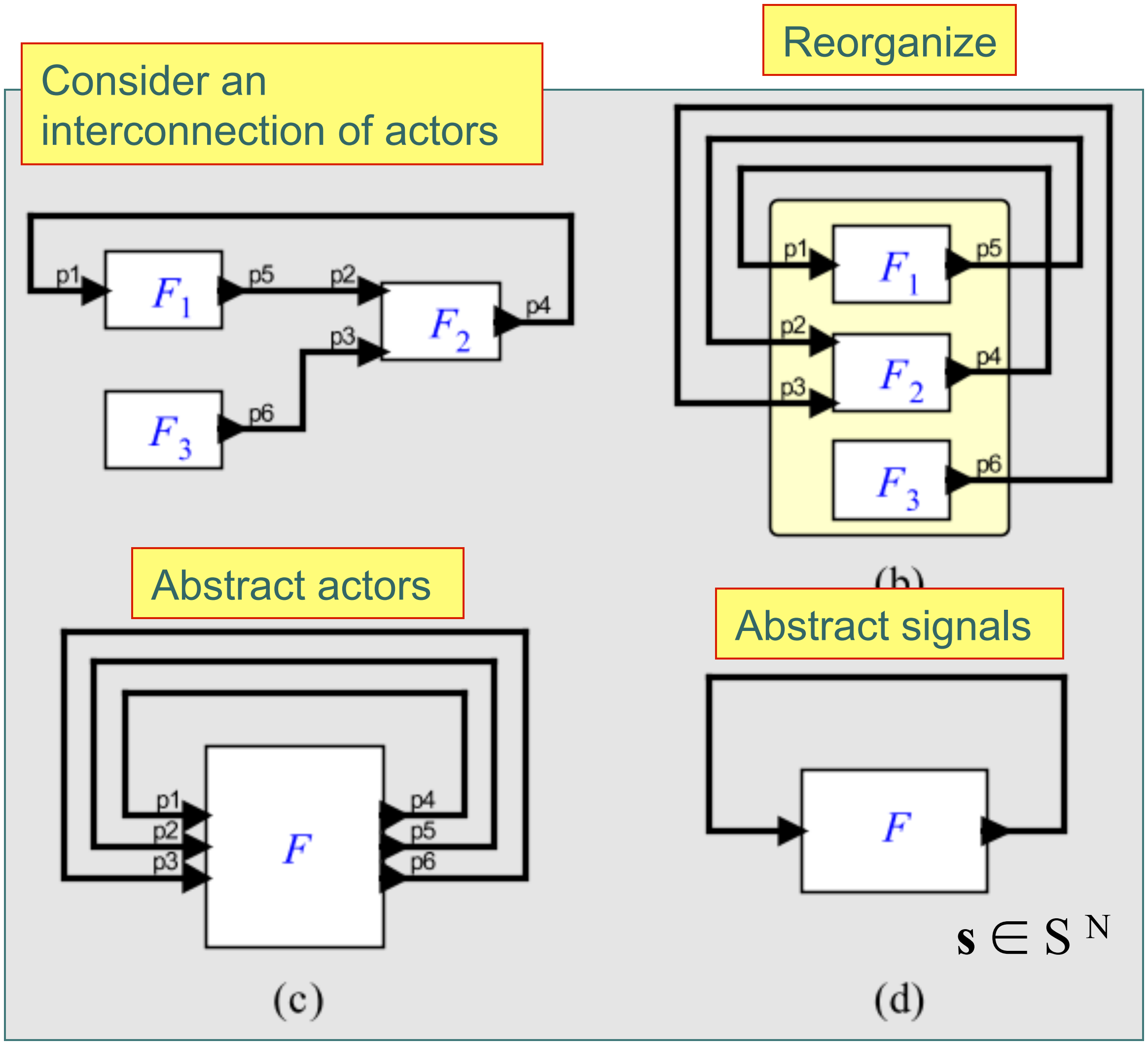
Observation: Any Composition is a Feedback Composition



If every actor is a function, then the semantics of the overall system is the least $\mathbf{s} \in S^N$ such that $F(\mathbf{s}) = \mathbf{s}$.

The behavior of the system is a “fixed point.”

Fixed Point Semantics



We seek an $\mathbf{s} \in S^N$ that satisfies $F(\mathbf{s}) = \mathbf{s}$.

Such an **s** is called a *fixed point*.

We would like the fixed point to exist and be unique. And we would like a constructive procedure to find it.

It is the *behavior* of the system.

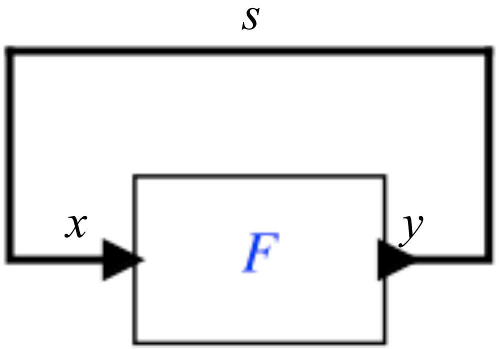
Data Types

As with any connection, we require compatible data types:

$$V_y \subseteq V_x$$

Then the signal on the feedback loop is a function

$$s\colon \mathbb{N} \to V_y \cup \{absent\}$$



Then we seek s such that

$$F(s) = s$$

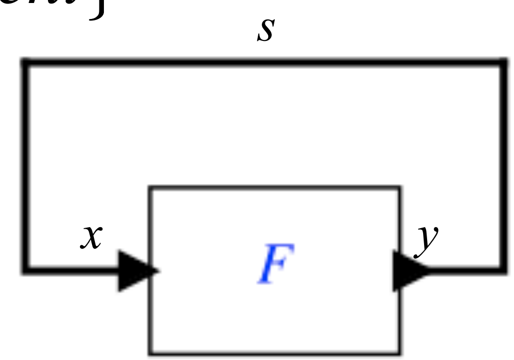
where F is the actor function, which for determinate systems has form

$$F\colon (\mathbb{N} \to V_x \cup \{absent\}) \to (\mathbb{N} \to V_y \cup \{absent\})$$

Firing Functions

With synchronous composition of determinate state machines, we can break this down by reaction. At the n-th reaction, there is a (state-dependent) function

$$f(n) \colon V_x \cup \{absent\} \to V_y \cup \{absent\}$$



such that

$$s(n) = (f(n))(s(n))$$

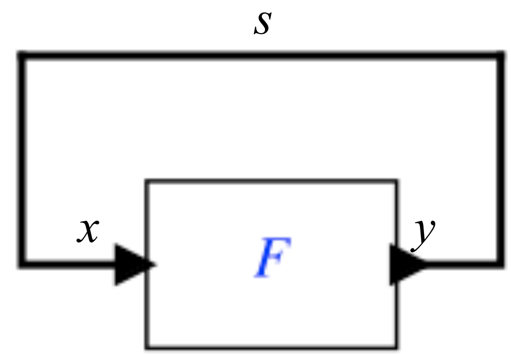
This too is a fixed point.

Well-Formed Feedback

At the n-th reaction, we seek $s(n) \in V_y \cup \{absent\}$ such that

$$s(n) = (f(n))(s(n))$$

There are two potential problems:

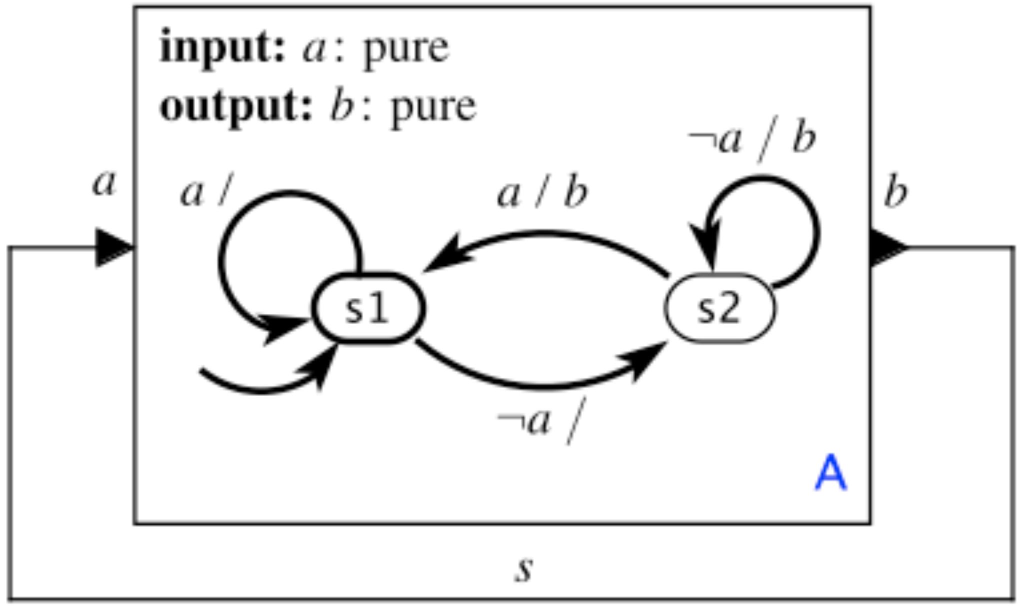


1. It does not exist.
2. It is not unique.

In either case, we call the system **ill formed**. Otherwise, it is **well formed**.

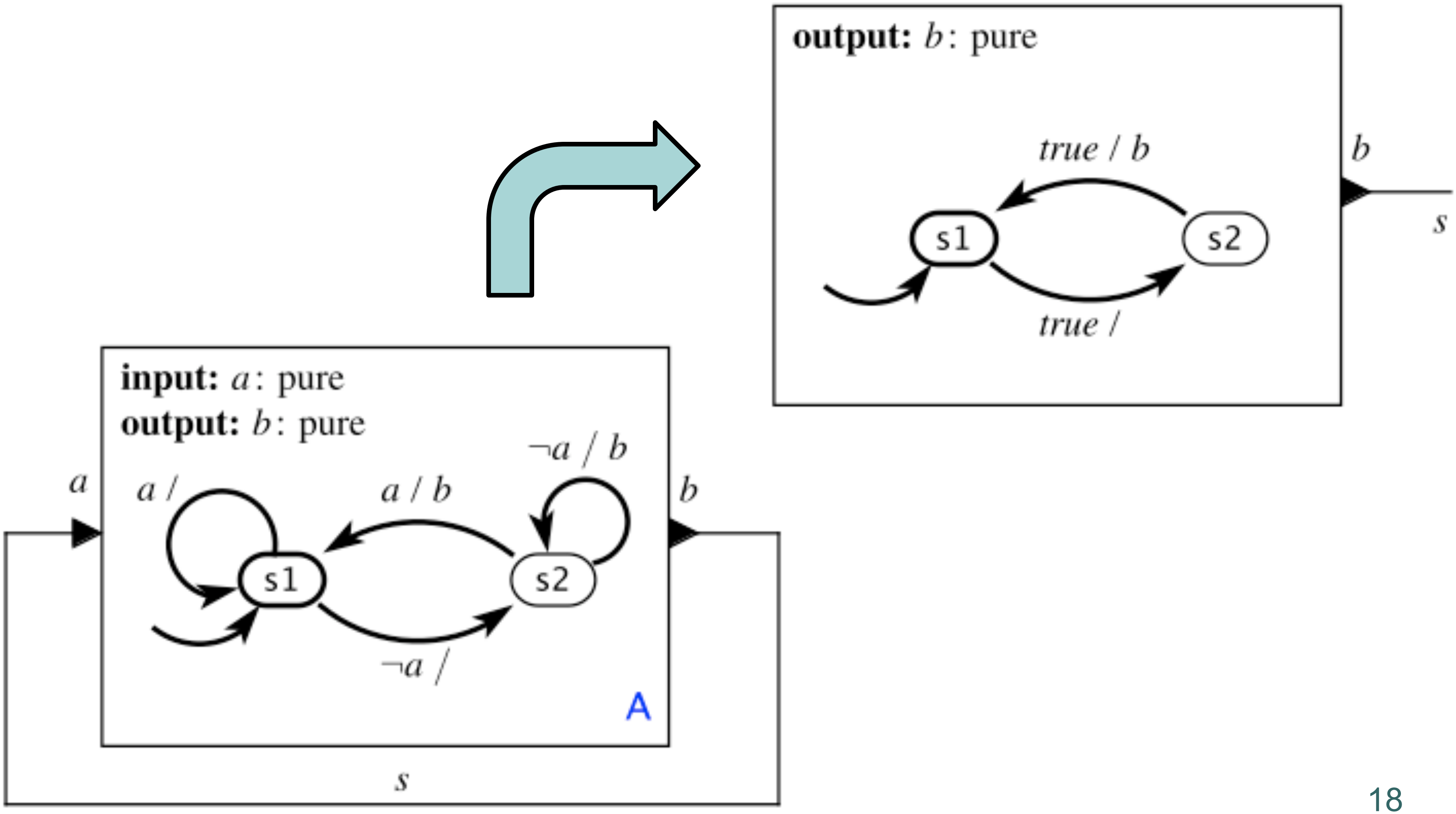
Note that if a state is not reachable, then it is irrelevant to determining whether the machine is well formed.

Well-Formed Example

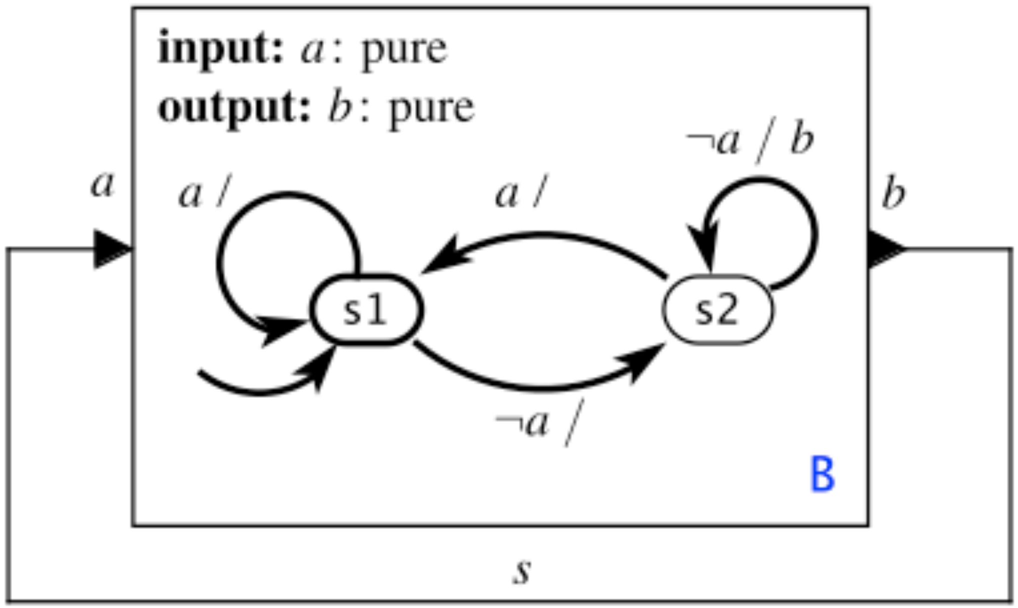


In state s1, we get the unique $s(n) = absent$.
In state s2, we get the unique $s(n) = present$.
Therefore, s alternates between *absent* and *present*.

Composite Machine

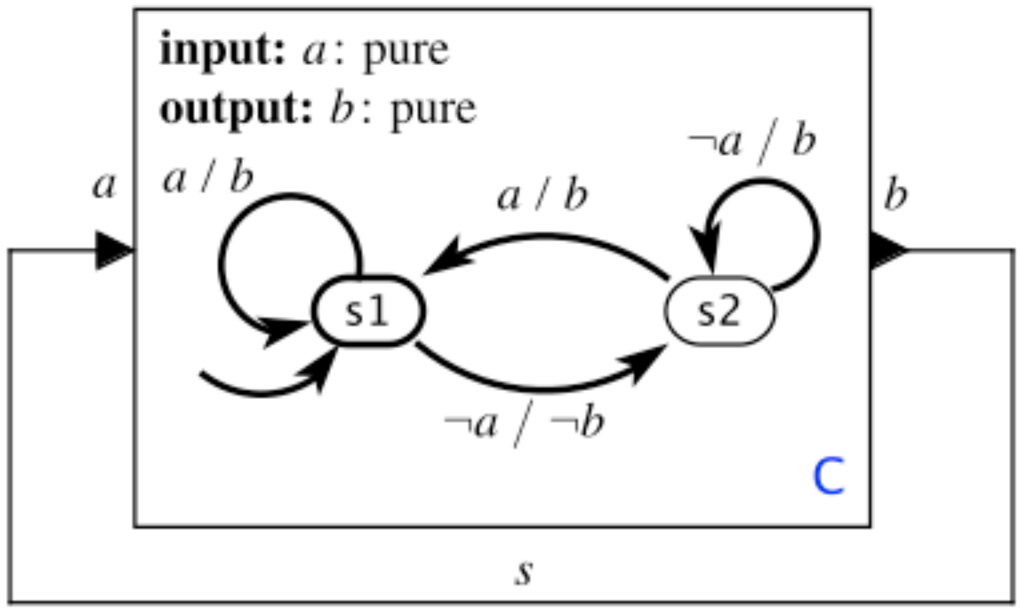


Ill-Formed Example 1 (Existence)



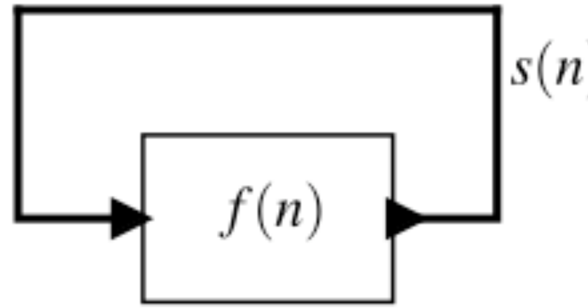
In state s1, we get the unique $s(n) = absent$.
In state s2, there is no fixed point.
Since state s2 is reachable, this composition is ill formed.

Ill-Formed Example 2 (Uniqueness)



In s1, both $s(n) = absent$ and $s(n) = present$ are fixed points.
In state s2, we get the unique $s(n) = present$.
Since state s1 is reachable, this composition is ill formed.

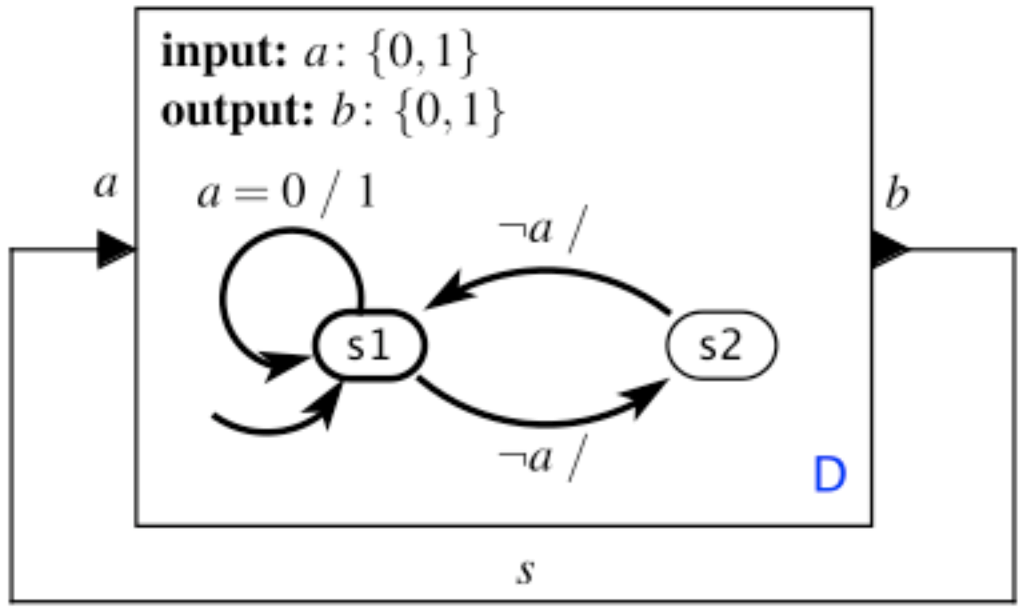
Constructive Semantics: Single Signal



1. Start with $s(n)$ *unknown*.
2. Determine as much as you can about $(f(n))(s(n))$.
3. If $s(n)$ becomes known (whether it is present, and if it is not pure, what its value is), then we have a unique fixed point.

A state machine for which this procedure works is said to be **constructive**.

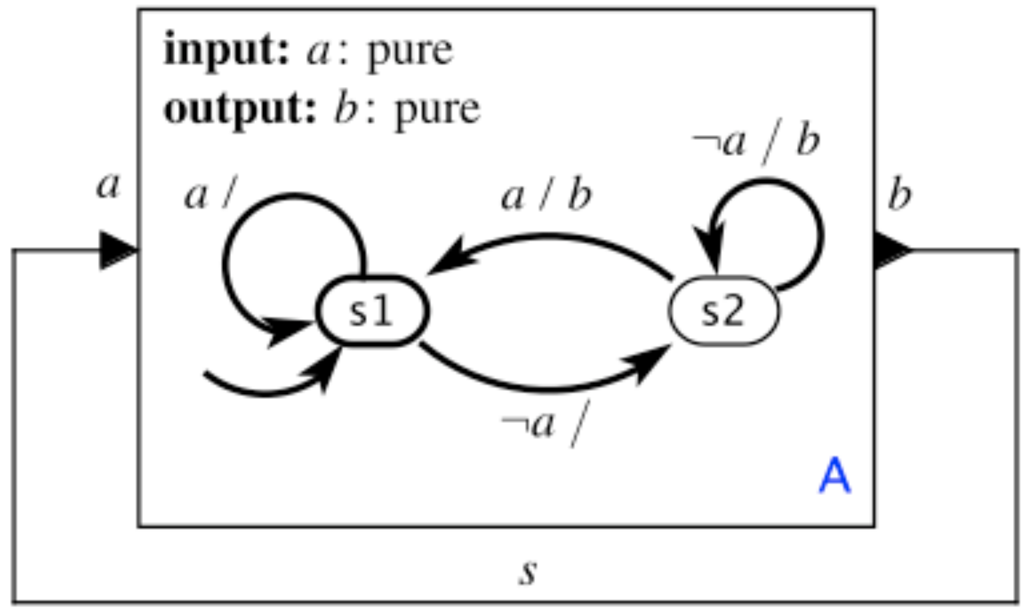
Non-Constructive Well-Formed State Machine



In state `s1`, if the input is unknown, we cannot immediately tell what the output will be. We have to try all the possible values for the input to determine that in fact $s(n) = absent$ for all n.

For non-constructive machines, we are forced to do **exhaustive search**. This is only possible if the data types are finite, and is only practical if the data types are small.

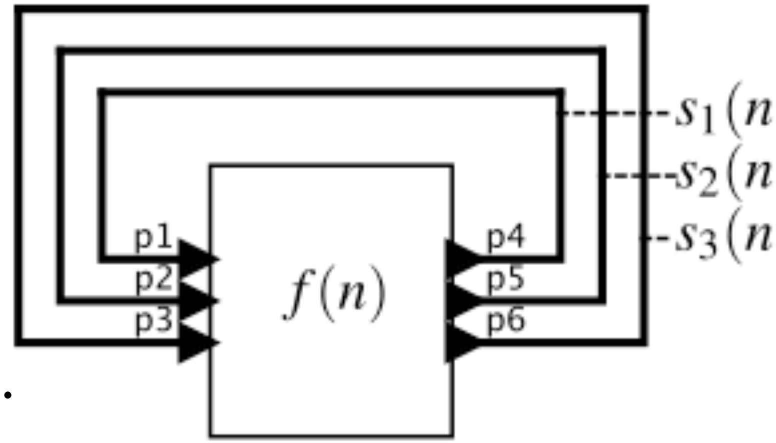
Must / May Analysis



For the above constructive machine, in state s1, we can immediately determine that the machine *may not* produce an output. Therefore, we can immediately conclude that the output is *absent*, even though the input is unknown.

In state s2, we can immediately determine that the machine *must* produce an output, so we can immediately conclude that the output is *present*.

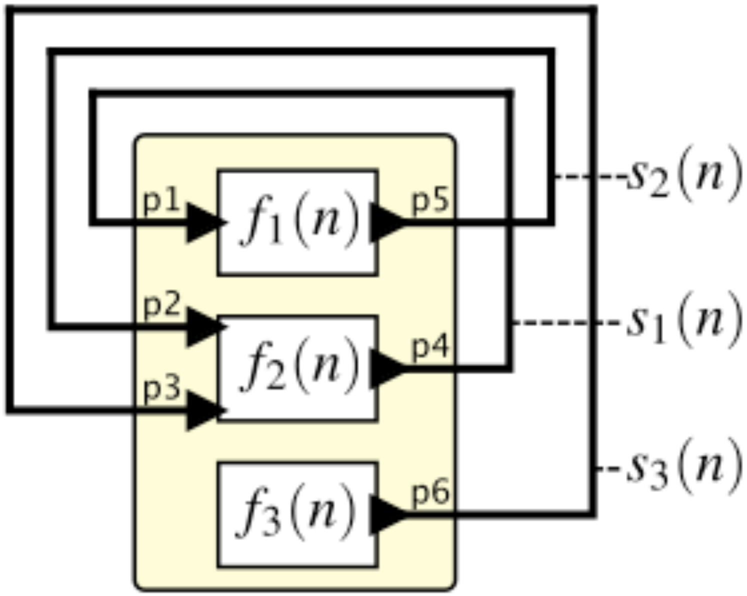
Constructive Semantics: Multiple Signals



1. Start with $s_1(n), \cdots, s_N(n)$ *unknown*.
2. Determine as much as you can about $(f(n))(s_1(n), \cdots, s_N(n))$.
3. Using new information about $s_1(n), \cdots, s_N(n)$, repeat step (2) until no information is obtained.
4. If $s_1(n), \cdots, s_N(n)$ all become known, then we have a unique fixed point and a constructive machine.

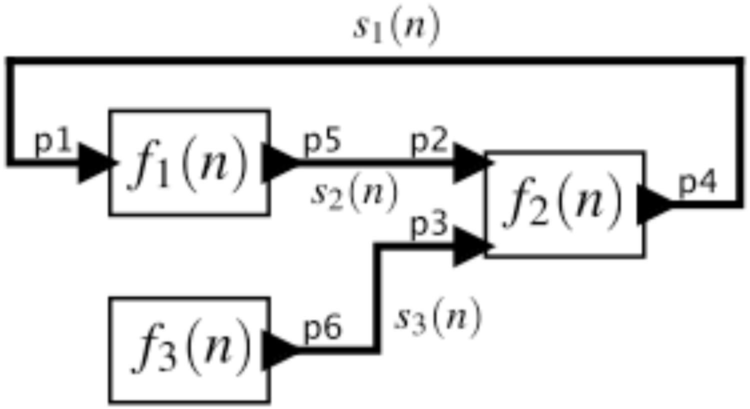
A state machine for which this procedure works is said to be **constructive**.

Constructive Semantics: Multiple Actors



Procedure is the same.

Constructive Semantics: Arbitrary Structure



Procedure is the same.

A state machine language with constructive semantics will reject all compositions that in any iteration fail to make all signals known.

Such a language rejects some well-formed compositions.

Conclusion

The emphasis of synchronous composition, in contrast with threads, is on *determinate* and *analyzable* concurrency.

Although there are subtleties with synchronous programs, all constructive synchronous programs have a unique and well-defined meaning.

Automated tools can systematically explore *all* possible behaviors. This is not possible in general with threads.